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Realistic real-time rendering of light shafts using blur filter: considering the effect of shadow maps

Realistic real-time rendering of light shafts using blur filter: considering the effect of shadow maps

Hatam H. Ali[1,2] & Mohd Shahrizal Sunar[1,2] & Hoshang Kolivand[3]

Abstract The ray marching method has become the most attractive method to provide realism in rendering the effects of light scattering in the participating media of numerous applications. This has attracted significant attention from scientific community. Up-sampling of ray marching method is suitable for rendering light shafts of realistic scenes, but suffers of consume a lot of time for rendering. Therefore, some encouraging outcomes have been achieved by using down-sampling of ray marching approach to accelerate rendered scenes. However, these methods are inherently prone to artifacts, aliasing and incorrect boundaries due to the reduced number of sample points along view rays. This research proposes a realistic real-time technique to generate soft light shafts by making use downsampling of ray marching in generating light shafts. The bilateral filtering is then applied to overcome all defects that caused by downsampling process to make a scene with smoothing transition while preserving on the edges. The contribution of this technique is to improve the boundaries of light shafts taking into account the effect of shadows. This technique allows obtaining soft marvelous light shafts, having a good performance and high quality. Thus, it is suitable for interactive applications.

1 Introduction

Light scattering in the participating media has become a popular rendering approach similar for rendering of light in virtual environments. In contrast, the effect of light represents an interaction that only occurs at the surface of the objects, the effects of light scattering are an interaction with suspended particles in air give rise to effects such as crepuscular rays, twilight, and light shafts [9, 12, 16, 20, 27]. The efficient rendering of these effects is a challenging task Forest and Segovia [14]. There are numerous of efforts have been spending to simulate the light shafts, but still suboptimal [18, 29, 32].

The natural scenes of light scattering depend on the number of phenomena such as emission, scattering, and absorption [4, 6, 26]. Therefore, the rendering equation requires taken into account these phenomena to evaluate light scattering. Consequently, the sampling can be computed the model of light scattering by determining points on lines that can sighted from light source. These points are tracked until hits a points of any object within scene and continuing until up to viewer.

Most methods that are used for generating light shafts adopt developed algorithms such as radiosity, ray tracing, and photon maps, but based on complex mathematical formulas. On the other hand, these methods did not care to an important issue, which is improvement the boundaries of light shafts to increase the realism. Furthermore, these research ignore use bilateral filtering that achieves a smooth transition with edges-preserving in generating the light shafts.

The light scattering is a sophisticated process as aforementioned above, which arises on the basis of emission of light from ubiquitous in space towards a viewer [17]. Thus, this process requires evaluating all points on the lines that can sighted from light source to be projection on the screen space with high resolution [33].

Ray marching method is most common than other proposed techniques that uses to generate light scattering. This method is computed light scattering by evaluating all points of the light source along view ray until up to the viewer Lin, et al. [21]. Therefore, several of methods have adopted this concept by using a trick to reduce of sample points along view rays to evaluate all points that potential seen from light source. Thus, the essential idea is to speed up rendering light scattering while avoiding evaluate all points along view ray that blocked of light source as much as possible. This research adopts a technique to generate efficiently and effectively light shafts using downsampling and bilateral filtering.

This technique consists from three important phases to generate accurately and efficiently light shafts. First, the sample points is reduced along view up less than a quarter compared to the requirements of brute force method. Second, shadow maps are improved to generate shadows which increase a realism of rendered scenes. Third, the bilateral filtering contributes in smooth-ing along boundaries of light shafts to remove artifacts due to reduce points of sample.

The immediate next section recapitulates related works. Section 3, explores the methodol-ogy for rendering the final scene. The results and related discussions are presented in section 4. Section 5 presents the conclusions and puts forward suggestions for future works.

2 Related work

Rendering a scattered light in participating media is a well-sought topic in computer graphics. Typically, it is tackled using ray marching technique to compute a contribution of light in each point along a ray, Pharr & Humphreys [24] exploited stochastic method to model scattering of participating medium. Several methods in domain of model scattering have put under control user in order to serve artistic requirements [2, 23]. Ament [2] proposed an interactive direct volume render technique to overcome problems that related with single light scattering and hard shadows using low-pass filtering. The unique characteristic for this technique is that smoothness of hard shadows in real time, but at the expense of realism. While Nowrouzezahrai [23] presented a technique to generate volumetric lighting by using photon beams method. The results of this technique closest to do in volumetric effects what professional artists do in hand drawn. However, this technique suffers lack for rendering in real time also do not take into account the realistic properties of participating media.

Many approaches have exploited a graphics hardware to obtain appropriate textures. Since the seminal work of Dobashi [10], some techniques have been proposed [7, 11, 12, 20, 29]. Most of these techniques focus on making the interactive rendering of typical light shafts caused by global illumination in atmospheric. Imagire [15] proposed a technique to remove from artifacts that resulting from rendering of volumetric effect using sampling hulls. Furthermore, it get rid from aliasing in shadows using interpolation. This technique valid of small regions within view ray. The previous technique Wyman [32] uses downscale to reduce points of rays, and determines ray marching regions to segments that viewed from light source.

A numerous of epipolar-based methods have shown interesting abilities for rendering volumetric lighting. The pioneer in this area Max [22] in his technique to display the light shafts based on volume shadows using epipolar slices. Engelhardt [13] proposed a technique that used geometry epipolar sampling and interpolated bilaterally to reduce the points of rays. Klehm [17] proposed a method that based on filtering process instead of the ray marching method, which depends on a rectification method for the image-based shadow linearly. Wyman [31] presented an algorithm to determine points of sample inside volumetric which viewed from light source using binary voxel grid in epipolar coordinates. Baran [3] presented incremental integration method using epipolar rectification to get good performance, Chen [8] in their method was extending to algorithm of [3], rely on 1D min-max mipmap.

Shin and Olano [25] presented a technique that focused on enhancing the edges of light shafts to get on naturally attenuation, also effect of particles depending on distributing of particle density precisely. If the participating media is not thick in this case it utilize from ray-marching to obtain reduce number of samples that lead to appear particle effects more obvious. However, Gabor's noise was used as an inputs to directions but the results was not convenient for viewer.

The main issue that related with state of the art in light shafts techniques is lack realistic in real time when use downsampling. Moreover, shadow maps still suffer from inherent defects in image-space shadows algorithms. This paper seeks to present a technique for improving the boundaries of shadow maps, smooth transition along and between edges of light shafts with preserving-edges, furthermore get rid of inherent artifacts in volumetric lighting caused by sampling in real time.

3 Method

In this section, a new technique is proposed for rendering the effect of soft light shafts that inspired from the concept soften hull of the light shaft. The downsampling is applied to reduce the number of samples over view ray of the volumetric light model. Based on this

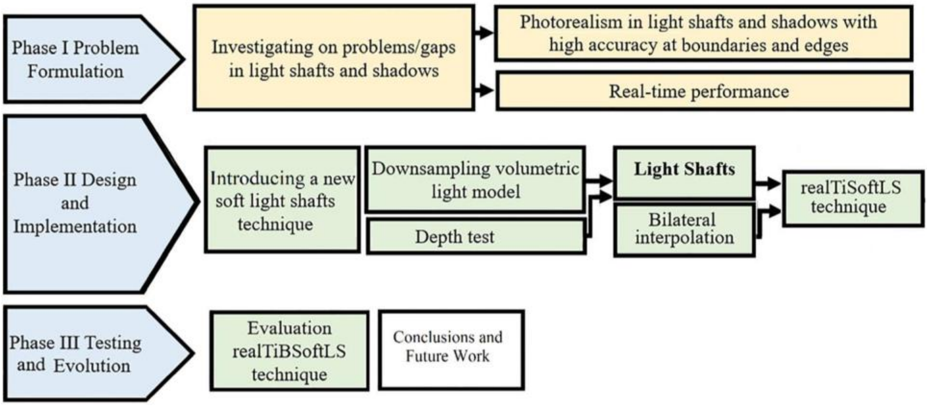

Fig. 1 the framework of realTiSoftLS technique

model, light shafts is rendered based on depth test of scene in high frame rate. As well, bilateral interpolation is used to improve the boundaries of light shafts and remove artifacts inherent in light shafts. Fig. 1 illustrates the framework of this technique, which consists of three phases.

The realistic real-time soft light shafts technique (realTiSoftLS) technique consists from two stages to generate light shafts accurately and efficiently. First, the sample points is reduced along view ray up less than a quarter compared to the requirements of brute force method. Second, the bilateral interpolation contributes in smoothing light shafts especially along boundaries and removes artifacts.

3.1 Downsampling volumetric light model

The model of single scattering is a better approximation for many participating mediums such as dust, smoke, etc. It is a simply redirect scattering once along view ray unto reaching to

viewer as illustrated in Fig. 2. A point in space that located on ray !ð Þ ¼ pc x !0 þ! wcis
!
defined by length units of ray c, direction w , and origin p 0 . The contrast is computed !ð Þ
values of the pixels from the light source L p w

transfer equation Wyman [25]. !; ! along ray as expressed by the radiance

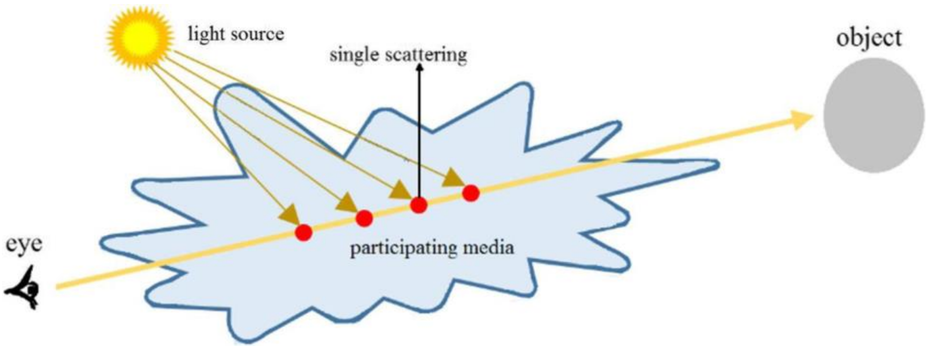

Fig. 2 single scattering on view ray from object to eye

$$\frac{dL\left(\vec{p}(c),\vec{w}\right)}{dc} = tL\left(\vec{p}(c),\vec{w}\right) + T\beta\int_{\Omega} L\left(\vec{p}(c),\vec{w}_0\right)F\left(\vec{w}_0,\vec{w}\right)d\acute{w} \tag{1}$$

Where T indicates transmittance to collision in a length units, t is a extinction coefficient, 췍 is the amount of intensity of light that remaining of scattering after collision, and $F\left(\vec{w}_0,\vec{w}\right)$ is the phase function denoting the light scattered with a probability in direction $\vec{w}$.

The above equation of integral and derivative is sophisticated to compute since the unknown radiance appears in derivative, normal, and integrated forms. Like equations can be computed by stochastic methods, but they are expensive for interactive applications. Thus, completely single scattering is adopted only, where indicates to in-scattering as following:

$$L_i\left(c,\vec{w}\right) = T\beta\int_{\Omega} L\left(\vec{p}(c),\vec{w}'\right)F\left(\vec{w}',\vec{w}\right)dw \approx L_i'\left(\vec{p}(c),\vec{w}\right) \tag{2}$$

Because the simplifying assumption, the resulting derivative equation

$$\frac{dL\left(\vec{p}(c),\vec{w}\right)}{dc} = tL\left(\vec{p}(c),\vec{w}\right) + L_i\left(\vec{p}(c),\vec{w}\right) \tag{3}$$

Can be computed analytically:

$$L\left(\vec{p}(c),\vec{w}\right) = e^{-Tc}L\left(\vec{p}_0+\vec{w}\right)\int_0^c L_i\left(\vec{p}(l),\vec{w}\right)e^{-T(c-i)} \tag{4}$$

Using approximated of a finite Riemann summation the right hand side:

$$L\left(\vec{p}(c),\vec{w}\right) \approx L\left(\vec{p}_0+\vec{w}\right)e^{-Tc} + \sum_{n}^{N} L_i\left(\vec{p}(l_n),\vec{w}\right)e^{-T(c-i)} \tag{5}$$

Where the step size is = c/N, which is inversely proportional to the number of sample points and is proportional to the length of the ray. For testing the depth, at first the depth distance between eye and models in a scene is stored as a 2D texture. Then a light source is rendered to texture using framebuffer object for both color buffer and depth buffer. In the final, program of the fragment shader is used to test the depth for each pixel on screen space.

3.2 Light shafts

p l w

sample rate of the points along view ray to reduce the rendering time. At start, the minimum of sample points is determined, which divides along distance of the view ray. The, for each point along view ray is computed light scattering which their product is gathered to the accumulated radial radiance. It is stored in the pixel spear-headed by the view ray.

The program of pixel shader considers the keystone in performing downsample process, where number of the samples is reduced to less of half. Some of parameters could be tuned to adjust the light scattering. It can be identified for example (Number of sample points, Decay, Density, Weight, Exposure, Occluding object FBO rendering resolution) as in Algorithm 1.

Algorithm 1: generating light shafts
Step 1. Segment = Current pixel position light
Step 2. Segment * = 1.0 / samples * density

```
Step 3. illuminationDecay = 1.0
Step 4. For i = 0; i < samples; i++
   textCoo - = Segment
   color = texture2D(firstPass, textCoo)
   color * = illuminationDecay * weight
   gl FraColor + = color
   illuminationDecay * = Decay
   End For
Step 5. return (gl FraColor * = exposure)
```

For rendering scene in a homogenous media, defines in-scattering along a view ray using segments of the ray that can be seen of the light source. On other hand, in heterogeneous medium (i.e., fog, cloud etc.) will needs also into minding in segments of the light traversal occurred in the participating media. In other word, what the segments of the ray that causes in-scattering to be illuminated in the medium and how much is the attenuation along these disconnected segments of the ray directions.

3.3 Bilateral interpolation

In this technique, bilateral filtering is used to smooth shadow edges correctly. The shadow edges can be smoothed using the bilateral filtering gracefully with take into account the depth discontinuity.

This filter is a non-linear bilateral filtering, which replaces the intensity value for each pixel in an image by weighted average of intensity values of neighbor pixels. Moreover, bilateral filtering which has an important characteristic takes the difference of intensities into consid-eration to preserve on edges. Conceptually of bilateral filtering is that two pixels are close to each other, whether they occupy nearby spatial location or they have nearby values within photometric range as Eq. 6:

$$g(p) = \frac{1}{W}\sum_{p_i \in M} f(p_i) G_{hr}(|f(p_i) - f(p)|) G_{hd}(|p_i - p|) \tag{6}$$

Vhere W is normalization as:

$$W = \sum_{p_i \in M} G_{hr}(|f(p_i) - f(p)|) G_{hd}(|p_i - p|)$$

The range kernel (G_{hr}) is used for blurring differences in colors, where contributes in reducing effect of the distant pixels to define the range filtering. While the domain kernel (G_{hd}) is used for blurring the differences in coordinates, where reduces the influence of pixel p with a color value to represent the domain filtering. Parameters (hr) and (hd) are measures to compute the filtering amount of image g for pixels location and values pixel respectively. In this method, the filtered image $g(p)$ based on the effect of the bilateral filter on the sampling for each pixel of the image. While (f) is the original input image to be filtered, the coordinates of the current pixel (p) to be filtered, and (M) is the mask centered in (p). The weight (W) is assigned using the spatial closeness and the intensity difference Tomasi [28]. In this method the weight is based on Gaussian distribution is computed as Eq. 7:

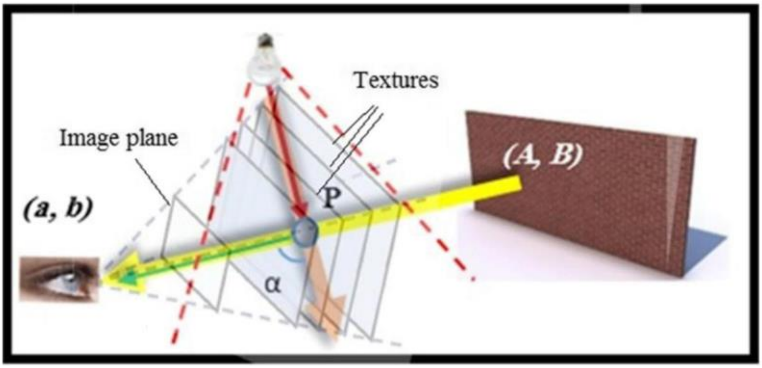

Fig. 3 light scattering model

$$G_h = e^{-hp^2} \quad (7)$$

Where h is a spatial extent of the kernel, size of the considered neighborhood, it must be adjusted to obtain comparable results. While (hr) is minimum amplitude of an edge, where sets amount of the colors desired to achieve combine of pixel values. That means both (hr) and (hd) can be controlled on results of the bilateral filtering. For instance, when (hr) increases by the bilateral filter, it becomes more similar to Gaussian blur due to the range Gaussian is flatter. While the domain parameter is increased (hd) becomes smoother.

In this technique, the mechanism of filtering is based on two 1D bilateral interpolation instead of 2D bilateral filtering to reduce of the cost. That means the two filters used one after the other, filter of each image row and then each column. When the color and depth of the central texel is determined, then moves by steps toward row and column to accumulate values in two directions the positive and negative respectively from a central texel, these steps are called taps. Results of this operation lead to smooth transitions in vertical and horizontal direction with preserved on crisp edges.

The multiplied weights of the bilateral filtering have important features due to when none of the weights is close to zero, no smoothing occurs. In addition, the bilateral filter divides the input image into a large-scale and small-scale component. The large-scale component considers anti-aliasing of the input, so that the main edges are preserved. While the small-scale component considered residual of the filter, which can be using as texture or noise by interpolated.

3.4 ReaTiShafts technique

The realTiSoftLS technique inspired from the concept soft hull of light shafts with crisp edges which seen under light scattering model. In this technique the light shafts resulting from single scattering in participating media is evaluated as minimum of sample points of light along view rays. Bilateral filtering are then used to improve the boundaries of light shafts and remove artifacts that inherently in light shafts. In addition, soft bilateral filtering shadows technique is used by Ali et al. [1] for improving shadow maps to obtain shadows more realistic and free of aliasing.

For rendering a scenes containing effect soft light shafts these require three impor-tant components are the light source, objects, and viewer as illustrates Fig. 3. Obvi-ously, the distance between the viewpoint (a; b) and the object position (A;B) in yellow color represents transmittance, as well a distance between the light source and point P in red color represents amount of radiance reached into P point. The distance between the viewer and point P in green color represents light transmittance between them. The angle between light ray and view ray at P point represents the phase function which determines a direction of the scattering. The virtual planes are placed in the front of the viewer' eyes to contribute in calculate the scattering of light. These virtual planes are a 2D texture used in the pixel shader. The algorithm of scattering carries out according to the following steps:

Algorithm 2: soft light shafts
Step 1. At each pixel determines the visible surface point and its reflected radiance
Step 2. It iterates along the ray from the surface to the camera making small steps
- in-scattering term $L_i \; p(\vec{l}, \vec{w})$ is computed
- decay $e^{-T(c-i)}$ from the sample point to the eye is obtained, and
- Their product is added to the accumulated

radiance Step 3. Applying bilateral filter
Step 4. The accumulated radiance is stored in the pixel spearheaded by the ray

3.5 Contribution of the work

The main contribution in this research is to present a new technique for rendering soft light shafts which based on volumetric lighting model, downsampling, and bilateral interpolation. This technique generates the boundaries of light shafts correctly as appears in real world. Further, it contributes in generating soft shadows that based on shadow maps algorithm. Therefore, this technique is efficient and effective to use in rendering the scenes of computer gaming, movies and also interactive applications.

4 Results and discussion

This approach is measured and implemented on a 2.5 GHz Intel(R)HD Core(TM) i5–3210 CPU using an ATI Radeon HD 7670 M Graphics 4000. SBFVS was written in OpenGL 3.3 and the shader was compiled with Shader Model (3.0). All scenes were tested at 800 × 600 resolution for rendering models. The results, comparisons and discussions are provided for the proposed soft light shafts (realTiSoftLS) technique. The experimental results depict of the proposed realTiSoftLS technique in both the efficient and effective.

4.1 Qualitative evaluation

For qualitative assessment of the current research exploits two methods which are the visual human inspection, and user study. Visual human inspection represents raw human sense, which is vision. The findings are compared with comparable works to figure out how effective our techniques when are compared them with previous works. While, user study is a questionnaire to sample of participants to determine the best results in terms of realism of rendered scenes in similar way as comparative study [19].

Fig. 4 shows the positions of light points along view ray to compute the light shafts which can be interpolating faithfully. In specific, light shafts make the participating media seamlessly visible due to light that is scattered towards the eye. Whenever the light shafts are nearing of the light source, light scattered captures the high frequencies of the intensity of light shafts.

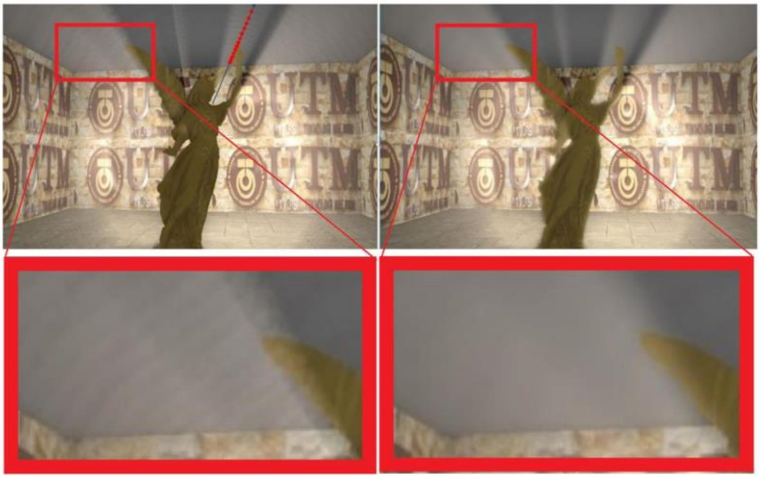

Fig. 4 (left) shows downsampling of ray marching for evaluating the light shafts the blue arrow refer to ray view and red points to samples, (right) the results of our technique using bilateral interpolation

Conversely whenever they are far away from the light source as shown Fig. 4 (left) the blue arrow refer to ray view and red points to samples, light scattered diminishes by capturing low frequencies of the intensity of the light shafts. This issue has been tackled in our technique using bilateral interpolation with respect to exploit downsampling strategy. This technique can be efficiently implemented on graphics hardware which demonstrates a good quality rendering at high frame rates as shown in the Fig. 4 (right).

Figure 5 comprised of four rows piled from top to bottom. The first row represents ray marching method (brute force) with upsampling up to 150 sample for spot light, point light, directional light respectively. The second row represents ray marching method with downsampling up to 32 sample for spot light, point light, directional light respectively. The third row represents ray marching method that filtered using Gaussian filter with downsampling up to 32 sample for spot light, point light, directional light respectively. The fourth row represents our technique with downsampling up to 32 sample for spot light, point light, directional light respectively. The high accuracy of light shafts in first row is as a reference image. Obviously, the second row contains aliasing artifacts of the light shafts is due to reduce number of sampling. The third row includes smooth light shafts, but suffer of blur all scene with a light leakage near the light source as well as the edges of light shafts appear distorted. While in fourth row the light shafts appear more realistic with smooth hull and meanwhile preserving on edges of light shafts specially the rims of the wall.

The resulting scenes from proposed technique asserts the performed well that have a realistic appearance with acceptable visual quality is due to the proposed technique soften of the light shafts while preserving its edges. Likewise, the scenes improved by using Gaussian filter have an unrealistic appearance, the outcomes are far from the reference images.

The Fig. 6 shows the result of our technique to interact realistic soft light shafts and soft shadows in the dynamic scene. This scene contains complex 3D models of the Dragon, Armadillo, Lucy and Elephant to render the effects that have been mentioned above. Obviously, the shadow resulting of this technique has stronger cues to create correctly physical shadow as shown in green and blue rectangles as shadow of the Lucy on wall and shadow of elephant ivory on Lucy. Moreover, whenever shadow was closer to the edge of object gives a detail relatively sharper while when it be far away gets softer. On the other hand, the soft volumetric shadows

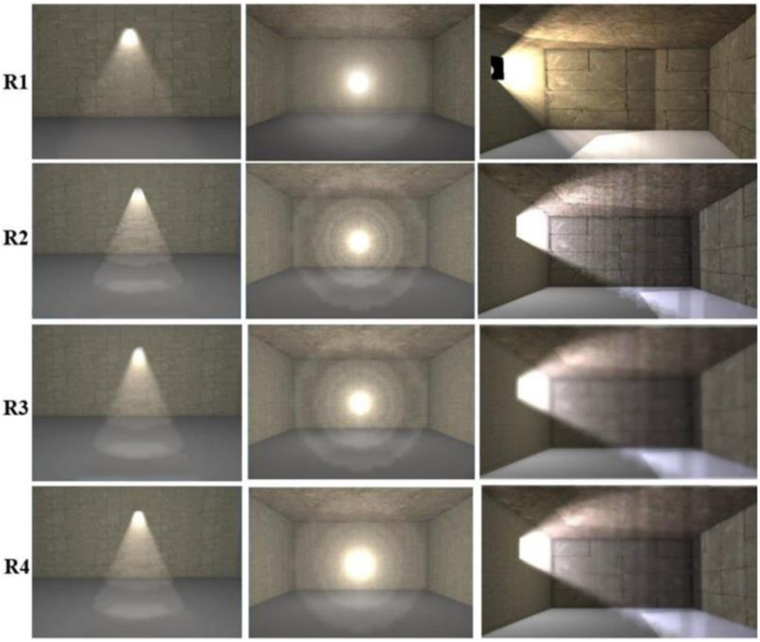

Fig. 5 the rows from top to bottom: R1) Brute force (ray marching method) with 150 sample; R2) ray marching with 32 sample; R3) downsampling with 32 blur by Gaussian filter; R4) our technique with 32 sample

that are generated have a crisp edges meanwhile the regions between these edges be smooth. As well as, same matter occurs when generating soft light shafts with crisp edges and the areas between these edges be seamless as shown in red and yellow rectangles.

The Fig. 7 shows different positions of light source to appear effects light shafts and shadows. In this Figure intentionally low sample rate is used to show the downsampling

Fig. 6 shows results of ReaTiShafts Technique structure that applied in this research. Near the light source there are the regularly spaced quad shapes which are initially placed ray marching points and additional points place at edges of depth. Fig. 7a, b, and c illustrate the effects light shafts and shadow when the light coming from outside to indoor for right, middle, and left a scene respectively. The number of samples that used in this scene is 64 samples.

Figure 8 illustrates the results evaluation of ReaTiShafts Technique by comparison a scene used downsampling without filtering. The image (top) in Fig. 8 represents ray marching method with sampling up to 50 sample. The image (bottom) in Fig. 8 represents our prototype development with 50 sample using bilateral filtering. The high accuracy of shadow, volumetric shadow, and light shafts in bottom image as a reference image. The top image suffers from aliasing artifacts of the shadow, volumetric shadow, and light shafts due to reduce number of sampling. Final image the shadow, volumetric shadow, and light shafts appear more realistic with smooth and meanwhile preserving on edges.

The user study in this research is conducted using questionnaire that consists from one question for ordered list of three images. The question focused on selecting the best image from three images according to realistic appearance [19]. Therefore, the participant selects the favorite one image from these three images, the best image is selected by using option button of the characters from A-C. The questionnaire have been conducted with size sample consisting of 30 persons participated 24 male and 6 female which they had an educational background in graphic design and computer science up to more than six years. This sample is selected in accordance to standard practices used in qualitative evaluation [19]. We used two computers in the same lab, the same monitor, and same conditions. The final images of results were not supported zooming and rotation.

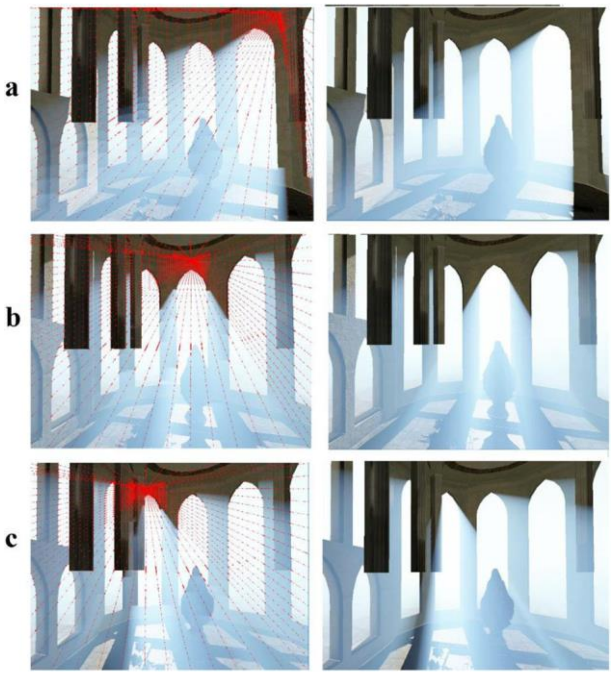

Fig. 7 shows downsampling of ray marching to appear effect of light shafts and shadow

Fig. 8 shows comparison between our results (bottom), and results of naïve method (top)

The Fig. 9 shows histogram of outcomes of sample evaluation from 30 person. Based on result of questionnaire 16 person voted in favor of our technique, 9 persons voted in favor of Real-Time VL technique [30], and 5 persons voted in favor of Anti-Aliasing LS technique [15]. The results from experiment show that 53% of the participants gave strongly positive responses to our technique in improving the realism due to yields of light shafts much better where edges crisper and regions softer than comparable techniques.

4.2 Quantitative evaluation

In the Fig. 10 illustrates comparison in performance between our technique, RSSRSM technique Klehm et al., [17], and Real Time VS technique Billeter et al., [5]. The rendered

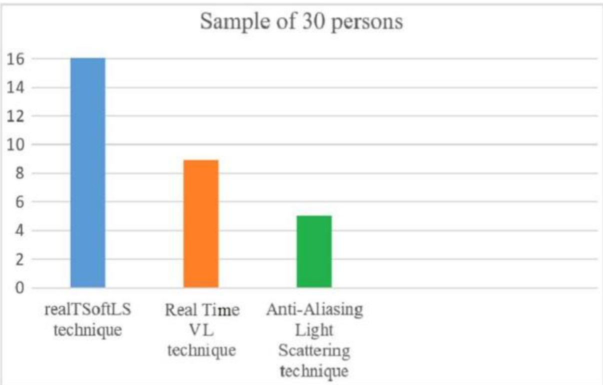

Fig. 9 graph for user study of our technique comparison with other techniques scene is of different 3D models of Stanford Bunny, Lucy, Dragon, and Buddha. The results obtained of performance are 74 fps, 72 fps, 69, and 60 fps for our technique of the Bunny, Lucy, Dragon, and Buddha respectively, the results of FRSSRSM technique of same models are 71 fps, 70, fps, 65 fps, and 55 fps, of the Bunny, Lucy, Dragon, and Buddha respectively, while the technique of Real Time VS the results are 63 fps, 62 fps, 57 fps, and 49 fps of the Bunny, Lucy, Dragon, and Buddha respectively, notice our technique faster than two previous techniques due to downsample and use deferred rendering technique in our implementation.

5 Conclusions and future work

This paper focused on augmenting the realism for virtual environments by generating soft shadows and soft light shafts. A combination of these effects is achieved to provide the cues depth of distance in the scenes, also gives relationship among objects that do not appear to be floating in space. This combination has been proven empirically to give a high performance and improve the realism for rendered scenes.

Nevertheless, in spite of noteworthy achievements in improving the quality and performance of shadows and light shafts, plenty of opportunities for the proposed techniques are still to be applied with various computer graphics applications. The proposed techniques can be further extended to tackle other species of effects of the light scattering such as caustics, aerial perspective, twilight sky color, smoke, fire, rainbow, halos, glories, and effects of underwater which are indispensable for generating realistic scenes of the natural phenomena. Furthermore, in future the results of these techniques can be applied in applications of augmented reality, applications of mobile devices, and in-studio effects for film making, etc.

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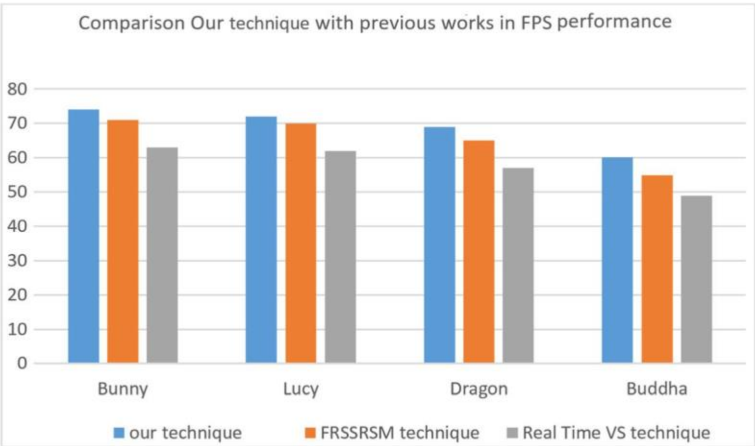

Fig. 10 Shows comparison in frame per second between our technique and previous methods

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