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Strategic asset allocation by mixing shrinkage, vine-copula and market equilibrium

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Abstract

We propose a new portfolio optimization method combining the merits of the shrinkage estimation (Jorion, 1985, 1986 and 1991), vine-copula structure (Aas and Berg, 2009), and Black-Litterman model (Black and Litterman, 1991 and 1992). It is useful for many investors to satisfy simultaneously the three investment objectives, estimation sensitivity, asymmetric risks appreciation and portfolio stability. A typical investor with such objectives is a sovereign wealth fund (SWF). We use China's SWF as an example to empirically test the method based on a 15-asset strategic asset allocation problem. Robustness tests using sub-samples not only show the method's overall effectiveness, but also manifest that the function of each component is as expected.

JEL classification: C11; C61; G11; G15; G17; G23

Keywords: Portfolio management; Estimation risk; Asymmetric risk; Views blending; Bayesian forecast

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1. Introduction

China Investment Corporation (CIC), the relatively young sovereign wealth fund (SWF) of China, has attracted much attention since its inception on 19 September, 2007. Due to the huge amount of foreign exchange reserves it can tap into, many are curious about its identity as an international investor, its investment objective and its strategic asset allocation (SAA). This specific instance and many others alike motivate us to find a portfolio optimization method to suit the demands for such long-term institutional investors' SAA decisions.

The three features of financial efficiency, good risk appraisal and allocation efficacy have intuitive importance to portfolio management, and therefore each of these aspects has been well developed. It is interesting to ask whether it is possible to combine the three elements together for investors with all three investment objectives simultaneously.

With respect to allocation efficacy, by which we mean stability of portfolio as well as level of diversification, the mean-variance analysis has been criticized. The most frequently applied solution is that proposed by Black and Litterman (1991, 1992) and further developed by He and Litterman (1999), and Satchell and Scowcroft (2000). They utilize the Bayesian rule to combine analysts' forecasts with the market equilibrium. This differs from the mean-variance method where the forecasts for every asset return are derived from the historic data. Based on the efficient market hypothesis, this method incorporates the market view as the basis for forecasting the future returns.

With respect to good risk appraisal for SAA, many papers discover the asymmetric dependence feature in asset returns (Longin and Solnik, 2001; Ang and Chen, 2002; Bae *et al.*, 2003; Hong *et al.*, 2007; Wang, *et al.*, 2013; Balla, *et al.*, 2014). Some assets are more likely to go down together, thus diminishing the effect of diversification. In the multivariate backdrop, asymmetric dependence, distinctive from asymmetry in marginal distributions, has also been proven to influence on asset allocation decisions (Boubaker and Sghaier, 2013; Leal and Mendes, 2013). In addition, the fat-tail feature means that extreme losses would be underestimated if the common Gaussian distribution were assumed, as in the mean-variance analysis. Therefore, the copula method is important for risk management in asset allocation. The application of copula method in financial series estimation is developing rapidly. In particular, the vine-copula (Aas, *et al.*, 2009) offers flexible tools to handle risk management in multivariate portfolio problems.

Another important issue in portfolio management is the 'estimation error'. Many papers attempt to deal with the estimation problem (Barry, 1974; Jorion, 1986, 1991; Pástor, 2000; Pástor and Stambaugh, 2000). This problem is closely related to the robustness of the optimal asset allocation and the accuracy of the model's predictability. Hence, proper treatment in this regard is expected to improve the overall financial performance of the portfolio management process. Estimation is the first stage in almost every portfolio optimization model. However, if the possibility of estimation error is neglected, using different sets of observations from the same distribution can often lead to different results as to the underlying distribution. In response to this issue, Jorion's (1991) shrinkage method is widely applied, and has been proved to be effective in many cases. We intend to incorporate this into our method and expect it to be able to improve the overall financial performance (profitability) in our case.

In the same field, several papers attempt to improve on asset allocation problems for the central banks in terms of the previous three aspects. Petrovic (2011) and Leon and Vela (2011) apply the Black-Litterman model for central banks. They recognize the potential of the Black-Litterman for allocation efficacy and combine the market equilibrium with investors' opinions. Barros Fernandes *et al*. (2012) use the Black-Litterman plus re-sampling techniques to deal with the estimation error. However, the re-sampling method is less intuitively appealing and less theoretically founded than the Bayesian method used in Jorion (1991) and others for estimation error. The method in their paper also lacks our copula risk appraisal ability. Another reason for choosing Jorion's shrinkage estimation over the re-sampling technique is that the estimation of vine-copula structures in high dimensional situations entails the high cost of computer power. In the re-sampling procedure, the repeated estimations of the copula parameters would take too much time to justify its advantage over the shrinkage estimation, presuming such an advantage does exist.

In the following sections, first in Section 2 the methodology is proposed and elaborated. In Section 3, we provide empirical analysis on the case of CIC, targeting the overall effectiveness of the method as well as the implications for each Bayesian combined component. In the final section, we conclude and point out limitations of this research.

2. Methodology

2.1 Bayesian linkage for three components

The three components we intend to incorporate in order to postulate a joint distribution are the above mentioned market equilibrium for robust portfolio, shrinkage estimation for estimation error and vine-copula for risk appraisal. It is important that these are connected in an intuitive manner. We are enlightened by the Black-Litterman approach for joining the market view and investors' views using the Bayesian theorem. The combination of the three components can be interpreted intuitively using the method and it is written as:

$$f\left(r \middle| r_{shrink}, \pi, \Sigma, \theta_{copula}\right) = \frac{f_1(r_{shrink}|r,\theta_{copula};\pi,\Sigma)f_2(r|\pi,\Sigma;\theta_{copula})}{\int f_1(r_{shrink}|r,\theta_{copula}\pi,\Sigma)dr} \quad (1)$$

In our theory of Bayesian connection for the three components, the prior distribution represents the market view of the returns. $f_2(r|\pi, \Sigma; \theta_{copula})$ is assumed to be Gaussian distribution with mean values as predicted by the market equilibrium. Based on the prior, the investor expresses her view conditional on the market view return from the prior. The return should follow a distribution with mean as the market prior and a copula dependence structure as estimated from data. This means that the investor assumes that in the long run the returns should return to the equilibrium, but it is possible that the returns would deviate from the equilibrium in a manner predicted by the short run copula dependence pattern, and the shrinkage estimated returns represent the deviated short run returns. The Bayesian theorem approach of combination of different views is theoretically founded, compared to Meucci's (2009 and 2010) more subjective Black-Litterman copula opinion pooling and the entropy minimization method.

2.2 Prior

The prior distribution expresses the market view. Its design is inspired by the Black-Litterman model for incorporating the market equilibrium. It assumes that the Capital Asset Pricing Model (CAPM) is established in the long run and the derivation of the equilibrium returns of the assets is a process of inverse optimization of the market portfolio. If we write $\Sigma = Cov(\mu, \mu')$ to be the covariance matrix of the risky assets, then:

$$\pi = \delta \Sigma w_m \tag{2}$$

where $\delta = (\mu_m - r_f)/\sigma_m^2$ is estimated from data. μ_m and σ_m^2 is the market index's mean and variance, and r_f is risk free return. Also estimated from the data is the covariance matrix $\boldsymbol{\Sigma}$, and the market weight w_m is known. The market equilibrium π is thus generated to be more robust than the sample estimates of the returns.

Actually in the inverse optimization problem, the Markowitz portfolio optimization can be achieved if and only if Equation (2) is established for any $\delta \geq 0$ (Bertsimas *et al.*, 2012). The determination of δ will determine the equilibrium excess returns π. In our model, δ is the market risk sensitivity and it is determined by the CAPM model to represent the market level of risk aversion. If the market is in certain form of market efficiency, the rationale for incorporating the market equilibrium returns is that the long-term equilibrium as a benchmark would forecast the future returns in certain degree and stabilize the asset allocation outcome.

2.3 Investor's View

The investor's view refers to the density distribution function, $f_1(r_{shrink}|r, \theta_{copula}; \pi, \Sigma)$. It contains the other two components of our model, the copula dependence and the shrinkage estimation of the returns. The incorporation of these two follows the Bayesian rule, and therefore the probability density function is a vine-copula function with parameters such as the copula coefficients, the return vector from the prior, r, and the shrinkage estimated returns r_{shrink} as function inputs.

For the estimation of r_{shrink}, the Bayesian-Stein method is described in Section 2.4. We follow Jorion (1986) and we have:

$$r_{shrink} = (1 - \widehat{w})\overline{Y} + \widehat{w}\overline{Y_0} \tag{3}$$

with

$$\overline{Y_0} = \frac{\mathbf{1}'\Lambda^{-1}\overline{Y}}{\mathbf{1}'\Lambda^{-1}\mathbf{1}}$$

$$\hat{w} = \frac{N+2}{(N+2) + (\bar{Y} - \bar{Y}_0\mathbf{1})'T\Lambda^{-1}(\bar{Y} - \bar{Y}_0\mathbf{1})}$$

$$\Lambda = \frac{\mathrm{T} - 1}{\mathrm{T} - \mathrm{N} - 2}\Sigma$$

(4)

where $\bar{Y}$ is the sample mean; $\boldsymbol{\Sigma}$ is the sample covariance matrix; T is the sample size and N is the number of returns.

In order to calculate the density function, we still need to determine the types and the parameters of the marginal densities of $f(x_k)$ and the bivariate copulas on each vine node for the C-vine structured dependence. ARMA – GARCH/APARCH – C-vine copula model combination is used for the task. The estimation contains two steps. In step 1 of the ARMA – GARCH/APARCH process, for each return series ARMA lag length parameters (u, v) are given choices from 0 up to 3. Two variance dynamics types are offered, GARCH and APARCH, with lag length parameters (p, q) also from 0 to 3. The residuals in the mean function are given choices from three types of distributions, namely Gaussian, Student-t and the skewed Student-t (Fernández and Steel, 1998). In the second step of the estimation process, each C-vine copula element is given the choice of 31 types of bivariate copulas. For both steps, the Akaike information criterion is applied for choosing the best fit models types, and maximized likelihood estimators are used for parameter values. Details of the ARMA – GARCH/APARCH – C-vine copula model combination can be found in Zhang *et al*. (2013).

However, for the purpose of incorporating the shrinkage return and the copula dependence, not all the results from the above two steps are needed. The copula parameters, θ_{copula}, derive from the estimation, but for the parameters in $f(x_k)$, the forecasted stationary mean values from the ARMA – GARCH/APARCH model are not needed. They should be based on the returns from the prior for compliance with the Bayesian assumption.

The above description is the objective reference model to incorporate the copula for asymmetric features. Subjective investor views can also be added in the same manner of the Black-Litterman model. The variance parameters of the marginal distributions, $f(x_k)$, can be multiplied by a parameter ranging from 0 to 1, representing investor's confidence in this view from 0 to 100 percent. Any linear combination of the individual returns can also form views like in Black-Litterman model. Since the dependency of the joint distribution is estimated by copula, the relationship between the linear combination views can be thus obtained easily.

2.4 Posterior

In Bayesian probability theory, it is always difficult to calculate a posterior distribution. For ease of applying the Bayesian theory, analytic posterior distributions are given when the prior and the likelihood function, i.e. $f_1(r_{shrink}|r, \theta_{copula}; \pi, \Sigma)$ in equation (1), take the forms of various usual continuous probability functions. These known analytic solutions of posterior and prior distributions are called conjugate distributions. However, in our case, in order to introduce the copula structure for better risk appraisal, the likelihood function is complex as well as flexible. The distribution function is a combination of marginal returns and copula dependence. In addition, there are 31 types of copula for each pair of returns in the vine structure and the number of types for each univariate return is 1536 (the product of 2 types of variance model, 3 different residual distributions, 4^4 combinations of ARMA-GARCH lag length parameters u, v, p, q). It is extremely difficult to obtain an analytic posterior.

Cheung (2009) introduced a simulation method for general Bayesian posterior distributions. A simulated posterior for equation (1) can thus be obtained in the following steps:

1. Prior distribution sampling. Sample $\{r^{(l)}\}_{l=1}^{L} \sim N(\pi, \Sigma)$, where L represents a large sample size, by applying the usual inverse probability integral transformation. The simulated distribution follows the prior distribution.

2. New probability vector calculation for the posterior distribution:

$$p^{(l)} = \frac{f_1\left(r_{shrink}|r^{(l)},\theta_{copula};\pi,\Sigma\right)}{\sum_{i=1}^{L} f_1\left(r_{shrink}|r^{(i)},\theta_{copula};\pi,\Sigma\right)} \tag{5}$$

3. The pair $\{p^{(l)}, r^{(l)}\}_{l=1}^{L}$ is the simulated posterior distribution with $r^{(l)}$ as a simulated value, $p^{(l)}$ is its probability.

It is worth noting that compared to a usual simulation applying the inverse probability integral transformation, the outcome pair $\left\{p^{(l)}, r^{(l)}\right\}_{l=1}^{L}$ here is different. For a usual simulation $\left\{r^{(l)}\right\}_{l=1}^{L} \sim N(\pi, \Sigma)$, it can be considered as a pair of $\left\{q^{(l)}, r^{(l)}\right\}_{l=1}^{L}$ where all $q^{(l)} = 1/L$, which means each $r^{(l)}$ is independent and equally important. This is not the case in the Bayesian posterior sampling. The proof of the above procedure can be found in Cheung (2009).

2.5 Portfolio optimization and performance assessment

The optimal asset allocation is solved based on the Bayesian distribution combining the above three components by maximizing an appropriate utility function. The chosen utility function must be able to reflect the investor's preference on higher moments other than mean and variance of the portfolio distribution and the asymmetric features of the assets' joint distribution. The Disappointment Aversion utility (DA utility hereafter) proposed by Gul (1991) is applied by Ang *et al*. (2005) and Hong *et al*. (2007) under asymmetric portfolio decisions similar to ours.

The DA utility is defined by the following equation:

$$DA(W) = \frac{1}{K}\left(\int_{-\infty}^{\mu_w} u(W)dF(W) + A\int_{\mu_w}^{\infty} u(W)dF(W)\right) \tag{6}$$

where $u(\cdot)$ is the felicity function in the form of CRRA utility here, i.e.

$$u(W) = \begin{cases} (1-\gamma)^{-1}\cdot(W)^{1-\gamma} & if\ \gamma \neq 1 \\ \ln(W) & if\ \gamma = 1 \end{cases}, \tag{7}$$

μ_w is the certainty equivalent according to the Constant Relative Risk Aversion (CRRA) power utility; $F(\cdot)$ is the cumulative distribution function of the wealth; and K is a constant scalar given by:

$$K = P(W < \mu_w) + AP(W > \mu_w). \tag{8}$$

The disappointment aversion parameter A in the above equations gives asymmetric preference on gains over losses. The risk preference parameter, γ, represents the investor's individual risk appetite, which is different from δ in Equation (2), the risk aversion of the market in the inverse optimization. We consider the risk preference $\gamma = 5$, and disappointment aversion $A = 0.45$ as appropriate levels representing China's SWF preference. The asset allocation is optimized by:

$$\max_{w} DA(W) \tag{9}$$

$$W = 1 + w'r \tag{10}$$

where the distribution of the asset returns r is modelled by the Bayesian method described previously.

For the purpose of assessing the optimal portfolio performance and the effectiveness of the Bayesian distributional method proposed in this paper, three dimensions of evaluation measures are devised, namely financial performance, risk predictability, and allocation efficacy. Financial performance is assessed by in-sample and out-of-sample DA utilities of the optimal allocation. Risk predictability is assessed by the difference between in-sample and out-of-sample skewness and the difference between in-sample and out-of-sample excess kurtosis. The allocation efficacy comprises the allocation diversification and stability, and these are evaluated respectively by the mean Herfindahl index, given by the sum of the squared asset weights as suggested in Barros Fernandes *et al*. (2012), and the average turnover given by the sum of changes of each asset between two consecutive years divided by the value of the portfolio.

3 Empirical Analysis

3.1 Data and comparison procedure

According to its annual report, financial assets account for the majority of CIC's investment portfolio, with public equities taking 32%, fixed-income securities 19.1%, and cash and others 3.8% as of 31 December 2012. Among the fixed-income securities investment, sovereign bonds of advanced and emerging economies account for 54.7% and 17.5% respectively, and another big chunk is investment grade corporate bonds, which takes 25.1%. Equity investment comprises three basic categories: US equities take 49.2%, other advanced economies equities 27.8% and emerging market equities 23%.

We follow these disclosed asset classes, using a total of 15 representative indices. For the fixed-income investments, six Bank of America Merrill Lynch Bond indices are selected. Four are sovereign bonds for advanced and emerging economies, while the other two are US corporate bonds and EMU AAA graded bonds. Six FTSE equities indices are used for the public equities investment, with three representing developed regions and three for the emerging economics. In addition to these 12 financial assets, there are three exchange-traded fund (ETFs) indices of real estate, oil and gold to represent the non-financial investments partially disclosed in CIC annual reports. Details of the indices are in Table 1.

Insert Table 1 around here

The data frequency is daily and the coverage period is from the beginning of 2006 until the end of 2012 to reflect the establishment time of CIC in 2007. A three-year rolling window approach of allocation optimization and evaluation is applied. This means that a three-year data window is used for the estimation of the next year's distribution and at the end of the next year the three-year window rolls a year forward to exclude the earliest year data and include the latest year data for the next estimation. The eight years' data coverage allows us to make such optimizations five times.

In addition to the Bayesian method comprising the market equilibrium, estimation errors and the copula risk appraisal techniques, there are four other estimation methods for comparison to manifest the advantage of our proposed method. These are listed in Table 2. Three of these methodologies exclude one of the three components. The purpose of this is that by comparison of the methods the effects of the missing component can be reflected. The fourth method is the simple sample mean-variance estimation as a benchmark. The five methodologies are compared across three dimensions: financial performance, risk predictability and allocation efficacy as described in section 3.5. It is worth noting that the third method, EsEq, is just the Black-Litterman model with the investor's views as the shrinkage estimated returns from the data.

Insert Table 2 around here

A robustness test of the proposed method is carried out after the initial comparison. This confirms the combination of the three components, and we then provide analysis of the optimal allocation outcome.

3.2 Comparison of methods

Table 3 displays the criteria statistics results according to the method described above. The investment universe contains all 15 asset classes across 5 years. The table shows the comparison of 10 criteria across the 5 methods. The first two criteria are the DA utilities of the optimal asset allocation according to a particular method. The in-sample DA utility is calculated based on the estimation using the data window. The out-of-sample DA utility is obtained by holding the optimal allocation from the estimation through the next year and using the daily data of that year as an empirical returns distribution. Although the difference in DA utility between methods seems small, but it does not mean the difference in allocation result is negligible. First, it is because daily returns are used in calculation rather than annualized version. Second, they are utility results rather than economic values.

The same logic for obtaining these in-sample and out-of-sample statistics applies in the skewness and excess kurtosis case. Differences between the in-sample and out-of-sample skewness and excess kurtosis are provided as criteria for the asymmetric risk prediction. It is because the consistency between in-sample and out-of-sample results represents the method's forecasting ability. The differences of these five methods are ranked later in increasing order to summarize their risk appraisal ability across various scenarios. Due to the nature of forecasting, in each single scenario performance of methods may be subject to chances but we are looking for the summarized result of many scenarios.

The remaining two criteria are the turnover and the Herfindahl index, to reflect allocation stability and diversification respectively. The turnover statistic needs the allocation information of the previous period, and therefore the values are zero in the first year. As to the out-of-sample statistics, data from next year are needed as the realized empirical distribution. Hence, in the last year there is no out-of-sample statistic. In the following analyses, the financial performance of a method is represented by the in-sample and out-of-sample DA utilities. The skewness and excess kurtosis differences are used as the criteria for the risk predictability. With regard to allocation efficacy, the turnover and the Herfindahl index reveal stability and diversification.

Insert Table 3 around here

However, it is difficult to determine the merits of each method, since there are many criteria and many years. For convenience in comparison, we have devised a ranking method for the statistics. The method is inspired by Barros Fernandes *et al*. (2012) in comparing their optimization method with the Black-Litterman. In their paper, the counts of scenarios for each method performed the best compared with other methods are reported in terms of several criteria. The statistics' unit is each performance criterion. In terms of a method overall performance across all the criteria interested, the comparison method cannot provide a synthesized view.

Our ranking method inherits and improves upon their comparison method. It looks at not only which method is the best, but also the other orders in performance rankings. So for example a second best method in one scenario would also generate a positive effect in its final performance statistic over all scenarios. Since the rankings can be combined across different criteria, it also overcomes the above drawback of lacking a synthesized view for a method. This additional feature is especially appropriate when in our optimization model we need to assess three dimensions – financial performance, risk appraisal ability and allocation efficacy.

The ranking contains two steps. In the first step, we rank the 5 methods based on the 6 criteria. For example, with respect to DA in-sample utility in 2008 the best utility method, EsEq, is ranked 1, and the worst method, Sample, has the lowest ranking, 5. The rank index for each distribution method is recorded for the six criteria we are interested in and across five years. Therefore, for each year from 2008 to 2012 there is a set of DA in-sample utility rank indices. In the second step, these rank indices across five years for the same criteria are summed and then ranked again from the smallest number summed to the largest. The smaller the number of the sum, the better the performance of this particular method in terms of a particular criterion, say the DA in-sample utility. It means that over the five years, this method has been ranked the highest overall. The procedure of this ranking method is demonstrated in Figure 1.

In Table 4, the six criteria are further summarized into three categories. Financial performance contains the DA in-sample and out-of-sample utilities. Its ranking is obtained by considering the two criteria as one. Similarly, risk predictability treats the skewness difference and excess kurtosis difference as one criterion, and allocation efficacy includes stability and diversification. The first column records the overall ranking covering the six criteria of each method.

It can be seen from the table that the proposed three-component method does perform best overall. It ranks second for financial performance and first for risk predictability. It confirms our prediction that the combination of copula for risk appraisal, market equilibrium for allocation stability and Bayesian-Stein for estimation error reduction outperforms other methods, i.e. those with only two components or the naked naïve MV analysis. The sample MV method only ranks second to last.

Insert Figure 1 around here

However, the result in Table 4 only contains five years. The merit of the three-component method may be just by chance. Also, the incorporation of the market equilibrium does not seem to improve the allocation efficacy. In contrast, the two methods without the market equilibrium are ranked first and second in this regard. To find out the reason for this, and to test the robustness of the proposed method, we continue with more analyses of the methods. In addition, the robustness test result in the following section can also tell us the effects of each of the three components proposed.

Insert Table 4 around here

3.3 Method robustness

In order to test for the robustness of the proposed method, we divide the data into four separations, and apply the same procedure as for method comparison. In addition to the 15 asset classes in section 4.2, there are three further asset allocation portfolios. We group the 12 financial assets together as the first separation. The second and third separations are six bonds as the fixed-income securities group and six stocks plus three commodity ETFs as the high risk securities group. In the following analyses we label these as bonds and stocks separations respectively.

Table 5 shows the overall rankings across the four separations. For each method there are 20 sub-rankings (4 separations times 5 years) summarized for the criteria of stability and diversification, while for financial performance and risk predictability there are 40 sub-rankings, because each of these contains two specific criteria. The table synthesizes all four situations and ranks the three-component method as best overall. The relative lack of performance in the allocation efficacy criterion leads us to reinstate its original two criteria format. In terms of stability, the proposed three-component method is ranked third. From the comparisons between the methods, the effects on stability of the three components, i.e. estimation error, copula and equilibrium, can be revealed. By comparing EsCoEq and CoEq, it is clear that the omission of estimation error has deteriorated the stability. Similarly, by observing the rankings in stability between EsCoEq and EsEq, and between EsCoEq and EsCo, it can be seen that the incorporation of copula has weakened the stability, whereas the equilibrium has strengthened it. In terms of the criterion of diversification, the first three methods with equilibrium incorporated have lower rankings, compared to the last two methods without. This is due to the fact that the market value weights of each asset class are not very averagely allocated.

Insert Table 5 around here

From Table 6 to Table 9, the specific rankings of the four separations are listed. The overall dominance of the three-component model is shown in Table 10. In the specifics here, we can see that the proposed model does not perform poorly in any of the situations. The result shows the robustness of the proposed model.

We have expectations when including each of the components, i.e. the estimation error, the copula or the market equilibrium, into the model. The copula should help with the risk prediction. The market equilibrium should be able to improve the allocation efficacy, and the estimation error should have a positive overall impact across the criteria of financial performance, risk predictability and allocation efficacy. The effects of each component can be revealed by comparing the three-component model with each of the two-component models. The two-component models each lack the effect of a particular missing component. Therefore the changes of rankings in each criterion are considered to be mainly due to the missing component. We use upward or downward pointing arrows beside the rankings of the three two-component models to indicate their changes compared with the proposed three-component model.

Table 10 is a summary of Tables 6 to 9. It groups the changes of rankings by the three two-component methods. If the ranking of a criterion is lowered, this means that the lack of a particular model component deteriorates the criterion performance, and thus proves the importance of that component. For the CoEq method, a combination of the copula and the market equilibrium, we expect that compared to the three-component model EsCoEq, it should manifest the characteristics of the estimation error factor. The incorporation of estimation error is supposed to improve the criteria in all three aspects systemically, and this is what we see in the result. In all four situations, the number of times a criterion ranking falls is higher than or at least equal to the number of times the ranking rises. For example, in the case of Bonds, all rankings decrease, which means improvement in all aspects. For Stocks, two rankings fall and two rise, which simply indicates that the benefits and disadvantages are balanced. Across all cases, if the estimation error factor is missing, more damage is done than benefit received. The effects of the other components, the copula for risk predictability and the market equilibrium for allocation efficacy, are more evident. The EsEq method demonstrates the copula impact whereas the EsCo shows the market equilibrium. In all four situations, all assets, financial assets, stocks and bonds, the inclusion of the copula component is proved to increase the risk predictability, and incorporating market equilibrium can improve allocation stability, as highlighted by the downward pointing arrows in bold text. These effects are unlikely to be by chance, due to their consistent presence in all four robustness testing situations. Other causalities, between copula and stability for example, might be false, and depend on the situation. Above all, the confirmation of our expectations for the three model components renders us confident in the model robustness and in its application for China's SWF strategic asset allocation decisions.

Insert Table 6 around here

Insert Table 7 around here

Insert Table 8 around here

Insert Table 9 around here

Insert Table 10 around here

4 Conclusion

This paper is motivated by the need for strategic asset allocation from ample funded, long-term institutional investors. China's SWF is taken as an example to illustrate a proposed three-component optimization method emphasizing both in long-term return and investment safety.

The method for forecasting the asset class returns combines three components, i.e. estimation error, copula and market equilibrium, using the Bayesian theorem, in order to deal with the well documented problems in mean-variance optimization, such as difficulty in estimating the proper parameters, lack of capability to handle non-Gaussian distributions, and the often extreme allocations. With regard to estimation error, Jorion (1985, 1986 and 1991) represents the direction of using Bayesian rule to incorporate the estimation risk. For the non-Gaussian returns, Hong *et al.* (2007) and other papers point out the importance of noticing asymmetries in individual assets and their dependence on the asset allocation decisions. In response to the unintuitive allocations of the mean-variance method, Black and Litterman (1991 and 1992) and subsequent papers propose models to incorporate the market equilibrium asset weights as a benchmark for analysis. We discover that a combination of the three is well suited to CIC's investment requirements on both returns and special attention to extreme risks.

In order to test for the effectiveness and the robustness of the proposed method, we rank it with other comparable methods in the three aspects most important to CIC: financial performance, risk management, and allocation efficacy. In various situations, the proposed three-component method gives the overall best performance. The effect of each component is also revealed through comparison to be as expected. Shrinkage estimation improves overall performance; vine-copula enhances risk appraisal; and market equilibrium improves allocation efficacy.

In the future research, improvements can be made in respect both of data and of methodology. With regard to the dataset utilized here, currently indices from FTSE and Merrill Lynch represent the financial asset classes around the world. However, if it were possible to use a customized set of indices reflecting the views of CIC's analysts, the allocation result would be more informative. The diversification decision and the relative importance of each asset class can provide more guidance as to the SAA decision. In the methodology aspect, the proposed model offering good financial performance, risk appraisal and allocation efficacy should be widely applicable in other asset allocation situations. For some insurance and pension management funds, as well as some university endowments, their SAA objectives resemble the investment-centred SWFs such as CIC. Therefore, the method should be tested in a wider range of applications, and with consideration of the performance in assets with different risk regimes and different durations. In addition, the robustness test can be enhanced further. A bigger dataset, longer horizon, and more data divisions should be attempted to confirm the proposition of wider applicability.

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Table 1 Data Source Description

Name	Type	Source	Mnemeric Code	Frequency
FTSE AW NORTH AMERICA	Stock Indices	Thomson Reuters Datastream	AWNAMR$(RI)	Daily
FTSE AW EUROPE			AWEROP$(RI)	
FTSE AW DEV ASIA PAC.			AWDVAP$(RI)	
FTSE EMERGING ASIA PAC.			AWAEPA$(RI)	
FTSE EMERGING LATIN AMER			AWAELA$(RI)	
FTSE AW MIDDLE EAST & AFRICA			AWMEAF$(RI)	
BOFA ML GLB GVT G7	Bond Indices		MLGGVG7(RI)	
BOFA ML USD EMRG SOV ASIA			MLIGDA$(RI)	
BOFA ML USD EM SOV LTN AM			MLIGDL$(RI)	
BOFA ML USD EMRG SOV EUR/ME/AFR			MLIGDE$(RI)	
BOFA ML US CORP AAA			MLC3ART(RI)	
BOFA ML EMU CORP LGE CAP AAA			MLELA0$(RI)	
ISHARES US REAL ESTATE	Commodity ETFs		U:IYR(RI)	
UNITED STATES OIL FUND			U:USO(RI)	
SPDR GOLD SHARES			U:GLD(RI)	

Notes: 'FTSE AW' refers to the FTSE all world indices. 'DEV' is short for developed countries. 'ASIA PAC.' is the abbreviation for Asian Pacific. 'BOFA ML' refers to Bank of America, Merrill Lynch. ‘Emerging countries’ is abbreviated to 'EM' or 'EMRG'. 'GLB', 'GVT', 'SOV', 'CORP', and 'LGE CAP' refer to global, government, sovereign bonds, corporate bonds, and large capitalization respectively. 'EUR/ME/AFR' refers to Europe, Middle East and Africa.
Source: Compiled by the author.

Table 2 Five Alternative Models

EsCoEq	Three-component model of Estimation Error, Copula and Market Equilibrium
CoEq	Two-component model of Copula and Market Equilibiurm
EsEq	Two-component model of Estimation Error and Market Equilibrium
EsCo	Two-component model of Estimation Error and Copula
Sample	Simple Mean-Variance model by historical returns

Source: Compiled by the author.

Table 3 Allocation Criteria across 5 Methods

	DAinsample	DAoutsample	skewinsample	skewoutsample	skewdiff	exkurinsample	exkuroutsample	exkurdiff	turnover	Herfindahl
2008										
	DAinsample	DAoutsample	skewinsample	skewoutsample	skewdiff	exkurinsample	exkuroutsample	exkurdiff	turnover	Herfindahl
EsCoEq	-0.11165	-0.11256	0.132312	0.085298	0.047014	0.169559	3.711341	3.541782	0	0.56849
CoEq	-0.11135	-0.11258	-0.01991	0.265078	0.284988	0.359389	3.89628	3.536891	0	0.62192
EsEq	-0.1115	-0.11269	0.018831	-1.44797	1.466803	0.021254	12.52276	12.50151	0	0.563322
EsCo	-0.1119	-0.11134	-6.28925	0.182246	6.471497	419.573	0.716303	418.8567	0	0.304174
Sample	-0.11175	-0.11197	0.001672	-0.45244	0.454111	0.002001	6.248988	6.246987	0	0.325619
2009										
	DAinsample	DAoutsample	skewinsample	skewoutsample	skewdiff	exkurinsample	exkuroutsample	exkurdiff	turnover	Herfindahl
EsCoEq	-0.11163	-0.11155	0.011059	-0.27598	0.287036	0.397809	0.234974	0.162835	0.640811	0.337697
CoEq	-0.11167	-0.11143	0.12662	-0.3342	0.460822	0.335493	0.370403	0.03491	0.908177	0.349243
EsEq	-0.11181	-0.11182	-0.00868	-0.23894	0.23026	-0.03189	-0.05582	0.023927	0.367264	0.465653
EsCo	-0.11159	-0.11136	-0.19485	-0.3683	0.173452	12.09569	0.920028	11.17566	1.082556	0.360958
Sample	-0.11183	-0.11142	0.000753	-0.28396	0.284711	-0.00942	0.645569	0.654988	0.458316	0.299011
2010										
	DAinsample	DAoutsample	skewinsample	skewoutsample	skewdiff	exkurinsample	exkuroutsample	exkurdiff	turnover	Herfindahl
EsCoEq	-0.11157	-0.11146	-0.17458	-0.55172	0.377147	0.221506	1.791427	1.569921	0.48821	0.243843
CoEq	-0.11171	-0.11152	0.09301	-0.51828	0.61129	0.285407	1.739939	1.454531	0.21619	0.299138
EsEq	-0.11451	-0.11377	-0.01043	-0.56461	0.554176	0.046608	2.393497	2.346889	1.659972	0.066667
EsCo	-0.11167	-0.11144	-0.21587	-0.62116	0.405295	4.437041	1.874159	2.562881	0.769485	0.230415
Sample	-0.11188	-0.11153	0.014467	-0.48701	0.50148	0.028068	1.495352	1.467285	0.06884	0.28655
2011										
	DAinsample	DAoutsample	skewinsample	skewoutsample	skewdiff	exkurinsample	exkuroutsample	exkurdiff	turnover	Herfindahl
EsCoEq	-0.11132	-0.11117	-0.04417	-0.04432	0.00015	0.307915	0.442138	0.134222	0.722214	0.344264
CoEq	-0.11122	-0.11109	-0.0556	-0.05689	0.00129	0.618172	0.710675	0.092503	1.52243	0.573431
EsEq	-0.11143	-0.1111	0.000862	-0.01005	0.010908	0.009243	0.757508	0.748264	1.317486	0.213773
EsCo	-0.1114	-0.1112	-8.73061	-0.00847	8.722145	845.5981	0.610053	844.9881	0.496482	0.208899
Sample	-0.11141	-0.11111	-5.14E-05	-0.0318	0.031751	-0.00984	0.470388	0.48023	0.795511	0.210576
2012										

	DAinsample	DAoutsample	skewinsample	skewoutsample	skewdiff	exkurinsample	exkuroutsample	exkurdiff	turnover	Herfindahl
EsCoEq	-0.11122	0	0.014964	0	0	0.080257	0	0	0.825921	0.249259
CoEq	-0.11121	0	-0.04542	0	0	0.084172	0	0	1.095843	0.228651
EsEq	-0.11133	0	-0.0129	0	0	0.029853	0	0	0.3518	0.246443
EsCo	-0.11126	0	-0.16954	0	0	2.040057	0	0	0.48981	0.242937
Sample	-0.11128	0	-0.00746	0	0	0.036474	0	0	0.481838	0.268293

Source: Compiled by the author.

Table 4 Performance Ranking in Three Categories

	All	Financial Performance	Risk Predictability	Allocation Efficacy
EsCoEq	1	2	1	4
CoEq	2	1	2	5
EsEq	5	5	3	3
EsCo	3	3	5	1
Sample	4	4	4	2

Notes: The numbers indicate the rankings of each method compared with other methods according to a specific criterion indicated by the column caption.
Source: Compiled by the author.

Table 5 Performance Rankings Summarized from Four Sample Separations

	All	Financial Performance	Risk Predictability	Stability	Diversification
EsCoEq	1	2	1	3	4
CoEq	2	1	2	4	5
EsEq	5	5	3	2	3
EsCo	3	3	4	5	1
Sample	4	4	5	1	2

Notes: The numbers indicate the rankings of each method compared with other methods according to a specific criterion indicated by the column caption. The result is reached by summarizing rankings across the four sample separations.
Source: Compiled by the author.

Table 6 Ranking indices for all 15 assets

	All	Financial Performance	Risk Predictability	Stability	Diversification
EsCoEq	1	2	1	1	4
CoEq	2	1(↑)	2(↓)	5(↓)	5(↓)
EsEq	5	5(↓)	3(↓)	3(↓)	3(↑)
EsCo	3	3(↓)	5(↓)	4(↓)	1(↑)
Sample	4	4	4	2	2

Notes: The numbers indicate the rankings of each method compared with other methods according to a specific criterion indicated by the column caption. The upward and downward pointing arrows represent the rising or falling of the method's ranking compared to the proposed three-component method in the first row.
Source: Compiled by the author.

Table 7 Ranking indices for 12 financial assets

	All	Financial Performance	Risk Predictability	Stability	Diversification
EsCoEq	1	2	2	1	1
CoEq	2	1(↑)	1(↑)	4(↓)	4(↓)
EsEq	4	5(↓)	3(↓)	2(↓)	5(↓)

EsCo	3	3(↓)	4(↓)	5(↓)	2(↓)
Sample	5	4	5	3	3

Notes: The numbers indicate the rankings of each method compared with other methods according to a specific criterion indicated by the column caption. The upward and downward pointing arrows represent the rising or falling of the method's ranking compared to the proposed three-component method in the first row.
Source: Compiled by the author.

Table 8 Ranking indices for stocks

	All	Financial Performance	Risk Predictability	Stability	Diversification
EsCoEq	3	3	2	4	5
CoEq	5	4(↓)	3(↓)	3(↑)	4(↑)
EsEq	4	5(↓)	4(↓)	2(↑)	3(↑)
EsCo	1	1(↑)	1(↑)	5(↓)	1(↑)
Sample	2	2	5	1	2

Notes: The numbers indicate the rankings of each method compared with other methods according to a specific criterion indicated by the column caption. The upward and downward pointing arrows represent the rising or falling of the method's ranking compared to the proposed three-component method in the first row.
Source: Compiled by the author.

Table 9 Ranking indices for bonds

	All	Financial Performance	Risk Predictability	Stability	Diversification
EsCoEq	2	1	1	3	4
CoEq	4	2(↓)	2(↓)	5(↓)	5(↓)
EsEq	3	5(↓)	4(↓)	2(↑)	3(↑)
EsCo	5	3(↓)	5(↓)	4(↓)	2(↑)
Sample	1	4	3	1	1

Notes: The numbers indicate the rankings of each method compared with other methods according to a specific criterion indicated by the column caption. The upward and downward pointing arrows represent the rising or falling of the method's ranking compared to the proposed three-component method in the first row.
Source: Compiled by the author.

Table 10 Components' effects

	Financial Performance	Risk Predictability	Stability	Diversification
CoEq (Missing Estimation Error)				
All Asset	(↑)	**(↓)**	(↓)	**(↓)**
Financial	(↑)	(↑)	**(↓)**	**(↓)**
Stocks	**(↓)**	**(↓)**	(↑)	(↑)
Bonds	**(↓)**	**(↓)**	**(↓)**	**(↓)**
EsEq (Missing Copula)				
All Asset	(↓)	**(↓)**	(↓)	(↑)
Financial	(↓)	**(↓)**	(↓)	(↓)

Stocks	(↓)	**(↓)**	(↑)	(↑)
Bonds	(↓)	**(↓)**	(↑)	(↑)
EsCo (Missing Market Equilibrium)				
All Asset	(↓)	(↓)	**(↓)**	(↑)
Financial	(↓)	(↓)	**(↓)**	(↓)
Stocks	(↑)	(↑)	**(↓)**	(↑)
Bonds	(↓)	(↓)	**(↓)**	(↑)

Notes: This table is a summary of the arrow indicators from the previous 4 tables.
Source: Compiled by the author.