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Group Consensus Measurement in MADM with Multiple Preference Formats

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ABSTRACT. *An approach is proposed for measuring the group consensus in the multiple attribute decision making (MADM) problems with experts' various preference information on alternatives. In the approach, multiple decision makers give their preference information on alternatives in different formats. The uniformities and aggregation process with fuzzy majority method are employed to obtain the social fuzzy preference relation on the alternatives. Accordingly, the ranking values of the alternatives are obtained based on the obtained individual expert's fuzzy preference relation and the social one. The group consensus can be measured based on the ranking values of the alternatives that are derived from the individual expert's preference information and the social one. An example of selecting robots is presented as an illustration.*

KEYWORDS: *Multiple Attribute Decision Making; Preference Information; Fuzzy Preference Relation; Ranking Values; Group Consensus*

1. Introduction

In Multiple attribute decision making (MADM) problems, alternatives are always evaluated against some noncommensurate and conflicting attributes. How to rank the alternatives or select the best one has attracted many researches (Chen and Hwang, 1992; Shih, 2005; Awasthi *et al*., 2007; Venkata Rao, 2008; Dalalaha *et al*., 2011; Ye, 2012; Liu *et al*., 2013). In MADM problems, the invited experts' preference information is often used to obtain the final selection of the best alternatives. However, the experts' judgments vary in form and depth. Different experts may use different ways when expressing their preference information on alternatives. The approaches to solving MADM problems can be classified into three categories according to the preference information given by the experts: (1) the approaches without preference information (Chen and Hwang, 1992; Awasthi *et al*., 2007; Dalalaha *et al*.,2011; Ye, 2012; Liu *et al*.,2013), (2) the approaches with information on attributes (Li, 1999; Weber, 1999；Xu， 2004; Xu, 2007; Chen and Niou, 2011; Yu *et al.*, 2013), and (3) the approaches with information on alternatives (Chen and Hwang, 1992; Chiclana *et al*., 1998; Malakooti and Zhou, 1994；Montero， 2008；Nurmi， 2008; Fan *et al.*, 2010; Wei *et al*., 2011).

This study falls into the third category with experts' preference information on alternatives, where several types of formats of preference information on alternatives are employed: preference orderings, utility values and fuzzy preference relations(Chiclana *et al*., 1998), linguistic term vector, normal preference relation, selected subset, fuzzy selected subset (Zhang *et al*., 2004) and pairwise comparison(Saaty，2008). Preference orderings of alternatives can be transformed into fuzzy preference relations (Chiclana *et al*., 1998; Xu， 2004). Also utility values of alternatives are always converted into fuzzy preference relations for ranking of alternatives (Chiclana *et al*., 1998). After the preference information from multiple experts are uniformed, fuzzy majority method with fuzzy quantifier can be used to aggregate these uniformed preference information into a social one and to select the best acceptable alternative (Chiclana *et al*., 1998).

Integration is an important task in decision support process (Chiclana *et al*., 1998; Rao, 2008; Liu *et al*., 2010; Perçina and Kahramanb, 2010; Vu *et al.*, 2014), as well as it does in group decision making process with preference information on alternatives (Cabrerizo *et al*., 2013; Chiclana *et al*., 2013). In group decision making process, multiple experts are always involved and express their preference information on alternatives in different formats due to different culture and education backgrounds, such as, fuzzy preference relations (Chiclana *et al*., 2013, pairwise comparison(Saaty, 2008), linguistics with different granularities (Mata, et al., 2009). Uniformities process and aggregation process are needed to determine the optimal alternative.

In addition to find the optimal solution to the MADM with multiple experts' preference information on alternatives, group consensus measurement is also an important issue in the process of decision making (Bryson, 1996; Kacpzrzyk *et al*., 1992; Herrera-Viedma *et al*., 2005; Mata *et al*., 2009; Xu and Cai, 2011; Chiclana *et al*., 2013; Cabrerizo *et al*., 2013).

With fuzzy preference relation being the basic format of the decision makers' opinions, Kacpzrzyk *et al*. (1992) propose an approach to calculating the group consensus based on the concept of fuzzy majority. The linguistic quantifiers are employed to represent a fuzzy majority. The group consensus is "soft" based on the fuzzy linguistic quantifiers, which is determined by the decision makers' subjective attitudes. In Brysons' study (1996), a method is proposed for measuring the consensus between two vectors by calculating the cosine value of their included angle. Herrera-Viedma *et al*. (2005) propose a support system model for reaching the consensus in group decision-making problems where experts express their opinions in linguistic preference relations with multiple granularities. By means of designing the basic linguistic term set, multigranular linguistic information is uniformed. A similarity function is defined to compute the consensus degrees amongst the experts (Herrera-Viedma *et al*., 2005). Mata *et al.* (2009) propose an adaptive consensus model for group decision making problems with experts' multigranular linguistic opinions. With fuzzy preference relation and multiplicative preference relations as the formats of experts' opinions respectively, Xu and Cai (2011) propose two methods for determine the weights of experts so that the group consensus be maximum respectively, i.e. goal programming model and quadratic programming model. Then two iterative algorithms are developed for group decision making to reach the consensus, respectively. By using the nonparametric Wilcoxon statistical test, Chiclana *et al*. (2013) present a detailed experimental study on comparing five most widely used distance functions for measuring the consensus in group decision-making problems. Cabrerizoet *et al.* (2013) analyze some prospects and open questions in applying consensus model based on soft consensus measures when dealing the group decision making (GDM) problems.

However, the group consensus measure approach to MADM with decision maker (DM)s' different formats of preference information on alternatives is not common. This paper proposes an approach to measuring the group consensus in MADM with DMs' preference information on alternatives in the formats of preference orderings, utility values, fuzzy preference information, linguistic term vector, selected subset, fuzzy selected subset, pairwise comparison on alternatives. The DMs' various preference information is normalized into fuzzy preference relation respectively, and aggregated the results into a social one based on fuzzy majority method with fuzzy quantifier; Then, the ranking values of the alternatives are assessed based on the obtained fuzzy preference relation and the social one; According to the ranking values of the alternatives, which are derived from the individual expert's preference information and the social one, the group consensus can be measured.

This paper is organized as follows: Section 2 describes the MADM problem with preference information on alternatives in various formats; Section 3 focuses on preference uniformities, where the different types of preference information on alternatives are transformed into fuzzy preference relation respectively. In section 4, preference aggregation and exploitation are conducted. Section 5 proposes the approach to measuring the group consensus. In section 6, an example is used to illustrate the proposed approach. Conclusion is given in section 7.

2. Problem description

The following assumptions or notations are used to represent the MADM problems:

- let $S=(S_1, \ldots, S_m)$, denote a discrete set of $m\,(\geq 2)$ possible alternatives.
- let $R=(R_1, \ldots, R_n)$ denote a set of $n\,(\geq 2)$ attributes.
- let $A=[a_{ij}]_{m\times n}$ denote the decision matrix where a_{ij} (≥ 0) is the consequence with a numerical value for alternative S_i with respect to attribute R_j, i=1, …, m, j=1, …, n.
- let E = (e_1, e_2, …, e_K) denotes the set of experts. Different experts can express their preference on the candidate alternatives in different formats, i.e., preference orderings, utility values and fuzzy preference information.

This paper considers the MADM problems with experts' preference information on alternatives in following formats.

- preference orderings can be used by an expert to express his preference on the alternatives: O^k=(O^k(1),…O^k(m)), where $o^k(\cdot)$ is a permutation function over the index set {1, …, m} and $o^k(i)$ represents the position of alternative S_i in the preference ordering, i=1, …, m. The alternatives are ordered from the best to the worst.
- utility values, or an utility vector can be used by an expert to express his preference on the alternatives: U^k =(u_1^k ,…, u_m^k), $u_i^k \in (0,1)$, $1 \le i \le m$, where u_i^k represents the utility evaluation given by the decision maker to alternative S_i .
- fuzzy preference information on alternatives can be given by an expert. The DM's preference relation is described by a binary fuzzy relation P in S, where P is a mapping $S \times S \to (0, 1)$ and p_{ik} denotes the preference degree of alternative S_i over S_k. We assume that P is reciprocal, by definition, (i) $p_{ik} + p_{ki} = 1$ and (ii) $p_{ii} = -$ (symbol '–' means that the decision maker does not need to give any preference information on alternative S_i), $\forall i, k$ (Chiclana *et al*., 1998).
- Let $L^k = (l_1^k, l_2^k, ..., l_m^k)$ be a linguistic term vector given by an expert e_k. Where, l_i^k is the linguistic evaluation by e_k to alternative S_i, i=1, …, m .
- Let $\overline{S} = \{S_{i_1}, S_{i_2}, ..., S_{i_m}\}$ be a selected subset of S by an expert, to express the preference on part of the alternatives. $\overline{S} \subset S$, $i_m < m$. Alternatives in $\overline{S}$ are equivalent and dominate those in the left of S . The alternatives in $S / \overline{S}$ are also equivalent to each other.
- Let $\tilde{S} = \{(S_{i_1}, l_{i_1}^k), (S_{i_2}, l_{i_2}^k), ..., (S_{i_n}, l_{i_n}^k)\}$, $i_n < m$, be a fuzzy selected subset of S used by an expert e_k, to express the preference on part of the alternatives using linguistic terms. $l_{i_j}^k$ is a linguistic term, $i_j = 1, ..., i_n$.
- pairwise comparison on alternatives: Let $H = (h_{ij})_{m \times m}$ be a pairwise comparison matrix used by an expert e_k. Where h_{ij} represents the ratio of the preference of alternative S_i to S_j and can be given in Saaty's 1-9 scale (Satty, 2008). Matrix H represents the following characteristics:

$$h_{ij} = \frac{1}{h_{ji}}, \qquad i, j = 1, ..., m; i \ne j \tag{1}$$

$$h_{ii} = 1, \qquad i = 1, ..., m \tag{2}$$

$$h_{ij} > 0, \qquad i, j = 1, ..., m \tag{3}$$

3. Preference uniformities

When multiple experts are involved in the decision process, usually two phases are needed to find the final solution: aggregation and exploitation. Aggregation is to combine opinions on alternatives from different perspectives; Exploitation is to rank the alternatives or to select the best one based on the social information on the alternatives. In this section, in order to aggregate the different formats of preference information on alternatives from multiple experts, the following types of preference information on alternatives are converted into fuzzy preference relations respectively.

3.1. *Transform preference orderings into fuzzy preference relation*

An expert can use preference orderings to express his or her preference on the alternatives. In this paper, the preference orderings would be transformed into fuzzy preference relations as follows (Chiclana *et al*., 1998):

$$p_{ij}^k = \tfrac{1}{2}\left(1 + \frac{o^k(j)}{m-1} - \frac{o^k(i)}{m-1}\right), \qquad 1 \le i \ne j \le m \tag{4}$$

3.2. ***Transform utility vector into fuzzy preference relation***

Also an expert can use an utility vector to express the preference on the alternatives. The utility vector can be transformed into fuzzy preference relations as follows (Chiclana *et al*., 1998):

$$p_{ij}^k = \frac{(u_i^k)^2}{(u_i^k)^2 + (u_j^k)^2}, \qquad 1 \le i \ne j \le m \tag{5}$$

3.3. ***Transform linguistic term vector into fuzzy preference relation***

In 1980, Yager proposes a method for transforming a fuzzy number into a centroid index. Cheng revise Yager's method (1998) as follows:

Given a trapezoidal fuzzy number $\tilde{C}$, denoted by (*a*, *b*, *c*, *d*, *w*). The membership function is defined as in (6). Function $f_{\tilde{C}}^L(x):[a,b] \rightarrow [0,w]$ is continuous and strictly increasing. Therefore, its inverse function exists, denoted by $g_{\tilde{C}}^L(x)$. At the same time, $f_{\tilde{C}}^R(x):[c,d] \rightarrow [0,w]$ is continuous and strictly increasing, and its inverse function also exists, denoted by $g_{\tilde{C}}^R(x)$. $g_{\tilde{C}}^L(x)$ and $g_{\tilde{C}}^R(x)$ are both continuous on (0, *w*).

The centorid point ($\bar{x}$, $\bar{y}$) of $\tilde{C}$ is defined as,

$$\bar{x}(\tilde{C}) = \frac{\int_a^b x f_{\tilde{C}}^L dx + \int_b^c x dx + \int_c^d x f_{\tilde{C}}^R dx}{\int_a^b f_{\tilde{C}}^L dx + \int_b^c dx + \int_c^d f_{\tilde{C}}^R dx} \tag{6}$$

$$\bar{y}(\tilde{C}) = \frac{\int_0^w y g_{\tilde{C}}^L dy + \int_0^w y g_{\tilde{C}}^R dy}{\int_0^w g_{\tilde{C}}^L dy + \int_0^w g_{\tilde{C}}^R dy} \tag{7}$$

Then the value of $\tilde{C}$ is obtained as,

$$value(\tilde{C}) = \sqrt{(\bar{x})^2 + (\bar{y})^2} \tag{8}$$

Chen (2001) proposes a simplified method of transforming a trapezoidal fuzzy number $\tilde{C}$ denoted by (*a*, *b* , *c*, *d*) into a numerical value as

$$D(\tilde{C}) = \frac{a+b+c+d}{4} \tag{9}$$

This simplified method is also applicable for the triangular fuzzy numbers since a triangular fuzzy number is a special case of a trapezoidal fuzzy number with b=c (Chen, 2001).

3.4. *Transform selected subset into fuzzy preference relation*

With a selected subset of S, e.g., $\overline{S} = \{S_{i_1}, S_{i_2}, ..., S_{i_m}\}$, the fuzzy preference relation on any two alternatives in S can be defined as,

$$p_{ij}^k = \begin{cases} 1, & if\ S_i \in \overline{S},\ S_j \in S/\overline{S}, \\ 0.5, & \text{otherwise}, \end{cases} \qquad i, j = 1, ..., m; i \neq j \tag{10}$$

3.5. *Transform fuzzy selected subset into fuzzy preference relation*

When an expert e_k gives fuzzy selected subset $\tilde{S}$ on S, for any two alternatives S_i and S_j, if they both belong to $\tilde{S}$, where $l_i^k = (u_i, \alpha_i, \beta_i)$ and $l_j^k = (u_j, \alpha_j, \beta_j)$, the fuzzy preference relation on them is,

$$p_{ij}^k = \frac{value(l_i^k)}{value(l_i^k) + value(l_j^k)}, \qquad i, j = 1, ..., m; i \neq j \tag{11}$$

where $value(l_i^k)$ and $value(l_j^k)$ are the values obtained from (8) or (9).
If none of S_i and S_j belong to $\tilde{S}$, then

$$p_{ij}^k = 0.5, \qquad i, j = 1, ..., m; i \neq j\ . \tag{12}$$

If S_i belongs to $\tilde{S}$ and S_j does not, then

$$p_{ij}^k = value(l_i^k), \qquad i, j = 1, ..., m; i \neq j \tag{13}$$

where $value(l_i^k)$ is the value obtained from (8) or (9).

3.6. *Transform pairwise comparison into fuzzy preference relation*

Suppose an expert e_k expresses a pairwise comparison matrix on S, $H = (h_{ij})_{m \times m}$. Then, the following formula can be used to transform $H = (h_{ij})_{m \times m}$ into a fuzzy preference relation (Zhang *et al.*, 2004)

$$p_{ij} = \frac{1}{2}(1 + \log_9 h_{ij}), \qquad i, j = 1, ..., m; i \neq j \tag{14}$$

4. Preference aggregation and exploitation

4.1. *Preference aggregation*

After the experts' preference information has been uniformed into fuzzy preference relations respectively, the next step is to aggregate these uniformed preference information into a social fuzzy preference relation. The social fuzzy preference relation can be obtained by using the ordered weighted averaging (OWA) operator to aggregate individual fuzzy preference relations (Yager, 1998). An OWA operator of dimension K is a function F as follows,

$$F:(0,1)^K \rightarrow (0,1) \tag{15}$$

In this paper, to aggregate p_{ij}^1 , p_{ij}^2 ,…, p_{ij}^K , F is associated with a weight vector $V=[v_1,v_2,...,v_K]$, where $v_h \in (0,1)$, h=1,…, K, and $\sum_{h=1}^{K} v_h = 1$. F can be expressed as

$$F(p_{ij}^1, p_{ij}^2,\ldots, p_{ij}^K)= V \cdot C^T = \sum_{h=1}^{K} v_h c_h \ , \quad i,j=1,...,m; i \neq j \tag{16}$$

where $C=[c_1,c_2,...,c_K]$ and c_h is the hth largest value among the collection of $p_{ij}^1, p_{ij}^2,..., p_{ij}^K$, $h=1,...,K$. $P^l=(p_{ij}^l)_{m\times m}$ is the matrix of the uniformed fuzzy preference relation on the alternatives from the expert e_l , l =1,…, K. The weight vector V can be obtained by a proportional quantifier Q (Yager, 1998), i.e.,

$$v_h = Q(h/K) - Q((h-1)/K), \qquad h=1,...,K \tag{17}$$

where Q is a fuzzy linguistic quantifier, e.g., "at least half" and "as many as possible".

If $p_{ij}^1, p_{ij}^2,..., p_{ij}^K$ are assigned importance $z_1,z_2,...,z_K$, respectively, and t_h is the importance associated with c_h correspondingly, $h=1,...,K$, then equation (17) is changed into follows:

$$v_h = Q\left(\frac{\sum_{l=1}^{h} t_l}{\sum_{l=1}^{K} t_l}\right) - Q\left(\frac{\sum_{l=1}^{h-1} t_l}{\sum_{l=1}^{K} t_l}\right), \qquad h=1,...,K \tag{18}$$

In this paper, semantics "most", involved in the fuzzy linguistic quantifier with a pair (0.3, 0.8), is used by the OWA operator to aggregate experts' individual preference relations, i.e.

$$G=(g_{ij})_{m\times m} \tag{19}$$

$$g_{ij} = F_Q(p_{ij}^1, p_{ij}^2,..., p_{ij}^K), \qquad i,j=1,...,m; i \neq j \tag{20}$$

where $F_Q(\cdot)$ is defined in equation (16) and Q is the fuzzy linguistic quantifier with "most" which is used to obtain the weight vector V in equations (16) and (17).

4.2. *Preference exploitation (Chiclana et al., 1998)*

4.2.1. *The quantifier guided dominance degree (QGDD)*

The QGDD is used to quantify the dominance that one alternative S_i has over the others in a fuzzy majority sense, i.e.,

$$QGDD_i = F_Q(p_{ij}^I, j=1,...,m; j \neq i), \qquad i=1,\ldots,m \tag{21}$$

where function $F(\cdot)$ is defined in equation (16). Q is a fuzzy linguistic quantifier defined in Chiclana *et al*. (1998).

4.2.2. *The quantifier guided non-dominance degree (QGNDD)*

The QGNDD represents the degree to which alternative S_i is strictly dominated by alternative S_j. It is calculated by

$$QGNDD_i = F_Q(1 - p_{ji}^N, j = 1,...,m, j \neq i), \qquad i = 1,...,m \tag{22}$$

where $p_{ji}^N = \max\{p_{ji}^I - p_{ij}^I, 0\}$.

4.2.3. *Selection policies of alternatives based on QGDD and QGNDD*

Given the QGDD and QGNDD of each alternative, the selection policy is composed of following three steps:

Step1. The alternative with the maximum dominance degree and the maximum non-dominance degree will be chosen as the solution to the decision problem, i.e.,

$$S^{QGDD} = \{S_i \mid S_i \in S, QGDD_i = \sup_{S_j \in S} QGDD_j\} \tag{23}$$

$$S^{QGNDD} = \{S_i \mid S_i \in S, QGNDD_i = \sup_{S_j \in S} QGNDD_j\} \tag{24}$$

where $QGDD_j$ and $QGNDD_j$ ($j = 1, \ldots, m$) are defined in (21) and (22), respectively.

Step 2. Conjunction selection policy.

Define $S^{QGCP} = S^{QGDD} \cap S^{QGNDD}$. If $S^{QGCP} \neq \phi$, then end. Otherwise continue to step 3.

Step 3. Sequential selection policy.

Dominance based sequential selection policy process QG-DD-NDD:

If $\#(S^{QGDD}) = 1$, then end. This is the selection set. Otherwise, define

$$S^{QG-DD-NDD} = \{S_i \mid S_i \in S^{QGDD}, QGNDD_i = \sup_{S_j \in S^{QGDD}} QGNDD_j\} \tag{25}$$

This is the selection set.

Non-dominance based sequential selection policy process QG-NDD-DD:

If $\#(S^{QGNDD}) = 1$, then end. This is the selection set. Otherwise, define

$$S^{QG-NDD-DD} = \{S_i \mid S_i \in S^{QGNDD}, QGDD_i = \sup_{S_j \in S^{QGNDD}} QGDD_j\} \tag{26}$$

This is the selection set.

5. Group consensus measurement

5.1. *Determine the ranking values of alternatives*

The essence of measurement method is to determine the ranking values of alternatives based on the individual fuzzy preference relation P^k ($P^k = (p_{ij}^k)_{m \times m}$) and the social one G ($G = (g_{ij})_{m \times m}$ from (20)). Based on the fuzzy preference relation P^k**,** the ranking values of the alternatives can be calculated as (Xu, 2004),

$$\alpha_i^k = \frac{1}{m(m-1)}(\sum_{l=1}^{m} p_{ij}^k + \frac{m}{2} - 1), \qquad i=1, \ldots, m \tag{27}$$

Thus, given the preference information from expert e_i, suppose Ind_i, $Ind_i = (\alpha_1^i, \alpha_2^i, ..., \alpha_m^i)$, i=1,…, K, be the ranking value vector of the alternatives derived from (27); In the same way, denote the ranking value vector of the alternatives from the social fuzzy preference relation G as

$$Soc = (\alpha_1^s, \alpha_2^s, ..., \alpha_m^s) \tag{28}$$

5.2. *Group consensus measurement*

Given two vectors, $T_i = (t_{i1}, t_{i2}, ..., t_{im})$ and $T_j = (t_{j1}, t_{j2}, ..., t_{jm})$, the consensus between these two vectors is (Bryson, 1996):

$$Con(T_i, T_j) = 1 - \text{sine}\,(T_i, T_j) \tag{29}$$

Thus, the consensus between the individual fuzzy preference relation P^k and the group (social) one G can be obtained based on the ranking value vectors derived from them, denoted as $Con\,(Ind_i, Soc)$. That is,

$$G\_Con = \frac{1}{K}\sum_{i=1}^{K} Con(Ind_i, Soc) \tag{30}$$

where $Con(Ind_i, Soc)$ is defined in (29).

6. Illustration

A robot user wants to select a robot and asks seven experts to help him make a decision. Four alternatives (i.e. S_1, S_2, S_3 and S_4) are provided for the user to choose. The attributes considered include: 1) R_1: costs ($10,000), 2) R_2: velocity (m/s), 3) R_3: repeatability (mm), 4) R_4: load capacity (kg). Among the four attributes, R_2 and R_4 are of benefit type, and R_1 and R_3 are of cost type. The decision matrix with the four attributes (R_1, R_2, R_3 and R_4) and the four alternatives (S_1, S_2, S_3 and S_4) is presented as follows:

$$A = \begin{pmatrix} 3.0 & 1.0 & 1.0 & 70 \\ 2.5 & 0.8 & 0.8 & 50 \\ 2.2 & 0.7 & 2.0 & 90 \\ 1.8 & 0.5 & 1.2 & 110 \end{pmatrix}.$$

Suppose the experts $e_1, e_2, ..., e_7$ provide their opinions on the four alternatives to help the user. They express their opinions as followings: e_1 gives a preference ordering, O^1={3, 1, 2, 4}. e_2 gives an utility vector, U^2={0.7, 0.9, 0.6, 0.3}. e_3 expresses a vector of linguistic terms, L^3=("fair", "good", "good", "very good"). e_4 presents a fuzzy preference relation matrix P^4. e_5 provides a selected subset {S_3, S_4}. e_6 gives a fuzzy selected subset {(S_2, "good"), (S_4,"very good")}. e_7 gives a pairwise comparison matrix on the four alternatives as follows: $H=\begin{pmatrix}1 & 1/7 & 1/3 & 1/5\\ 7 & 1 & 3 & 2\\ 3 & 1/3 & 1 & 1/2\\ 5 & 1/2 & 2 & 1\end{pmatrix}$. Using the normalization functions above, the uniformed fuzzy preference relation matrices from these experts are obtained respectively:

$$P^1=\begin{pmatrix}- & 1/6 & 1/3 & 2/3\\ 5/6 & - & 2/3 & 1\\ 2/3 & 1/3 & - & 5/6\\ 1/3 & 0 & 1/6 & -\end{pmatrix},\ P^2=\begin{pmatrix}- & 49/130 & 49/85 & 49/58\\ 81/130 & - & 81/117 & 0.9\\ 36/85 & 36/117 & - & 0.8\\ 9/58 & 0.1 & 0.2 & -\end{pmatrix},$$

$$P^3=\begin{pmatrix}- & 0.3077 & 0.3077 & 0.2\\ 0.6923 & - & 0.5 & 0.36\\ 0.6923 & 0.5 & - & 0.36\\ 0.8 & 0.64 & 0.64 & -\end{pmatrix},\ P^4=\begin{pmatrix}- & 0.4 & 0.3 & 0.4\\ 0.6 & - & 0.5 & 0.7\\ 0.7 & 0.5 & - & 0.8\\ 0.6 & 0.3 & 0.2 & -\end{pmatrix},$$

$$P^5=\begin{pmatrix}- & 0.5 & 0 & 0\\ 0.5 & - & 0 & 0\\ 1 & 1 & - & 0.5\\ 1 & 1 & 0.5 & -\end{pmatrix},\ P^6=\begin{pmatrix}- & 0.25 & 0.5 & 0\\ 0.75 & - & 0.75 & 0.36\\ 0.5 & 0.25 & - & 0\\ 1 & 0.64 & 1 & -\end{pmatrix},$$

$$P^7=\begin{pmatrix}- & 0.1093 & 0.2709 & 0.1782\\ 0.8907 & - & 0.7517 & 0.6300\\ 0.7291 & 0.2483 & - & 0.3599\\ 0.8272 & 0.3700 & 0.6401 & -\end{pmatrix}.$$

The OWA operator with fuzzy linguistic quantifier "most" is used to aggregate the eight experts' opinions, with the corresponding weight vector being $(0, 0, 0.15, 0.25, 0.25, 0.25, 0.1, 0)^T$. The social fuzzy preference relation matrix is obtained as,

$$G=\begin{pmatrix}- & 0.3261 & 0.3553 & 0.2989\\ 0.6739 & - & 0.5968 & 0.6729\\ 0.6447 & 0.4032 & - & 0.5439\\ 0.7011 & 0.3271 & 0.4561 & -\end{pmatrix}.$$

Based on the exploitation method in 4.2, QGDD of the alternatives are calculated: d_1=0.3028, d_2=0.6527, d_3=0.5131, d_4=0.3380, which is normalized into (0.1676, 0.3613, 0.2840, 0.1871). The selection result of the alternatives is S_2.

QGNDD of the alternatives are calculated: d_1 =0.6416, d_2 =1.0000, d_3 =0.9484, d_4=0.8493, which is normalized into (0.1865, 0.2908, 0.2758, 0.2469). The selection result of the alternatives is S_2. Based on the selection results from QGDD and QGNDD, the best alternative selected is S_2.

The vector of the ranking values of alternatives from every expert's preference are obtained as: Ind_1=(0.2222, 0.3333, 0.2778, 0.1667), Ind_2=(0.2749, 0.3096, 0.2526, 0.1629), Ind_3 =(0.1929, 0.2544, 0.2544, 0.2983), Ind_4 =(0.2167, 0.2750, 0.2917, 0.2167), Ind_5 =(0.1667, 0.1667, 0.3333, 0.3333), Ind_6 =(0.1875, 0.2800, 0.1875, 0.3450), Ind_7=(0.1715, 0.3144, 0.2364, 0.2781), which is showed in table1.

The vector of the ranking values of alternatives from the social fuzzy preference relation matrix is calculated as: *Soc*=(0.2067, 0.2870, 0.2576, 0.2487), which is showed in table1.

The consensus index between the individual ranking value vectors for every expert is calculated and the results are shown in table 2. In addition, the consensus index between the ranking value vector from the individual expert's preference and the social one is calculated and the results are shown in table 3. The group consensus index is 0.815.

7. Summary

This paper proposes an approach to measuring the group consensus in MADM problem with multiple types of preference information on alternatives. The approach is composed of three steps: 1) normalize the DMs' different formats of preference information into fuzzy preference relation respectively, and aggregate the results into a social one; 2) figure out the ranking values of the alternatives based on the fuzzy preference relation obtained from the individual DM's preference information and the social one; 3) measure the group consensus according to the ranking values of the alternatives, which are derived from the individual DM's preference information and the social one. The proposed approach is an extension for the current group decision making methods for consensus measurement by employing multiple formats of preference information from the experts. In addition, the group consensus measurement method proposed is innovative. The proposed approach is computationally simple, rational and can readily be incorporated into a computer-based system.

This paper is not without limitation. In this paper, seven formats of preference information are considered, i.e., preference orderings, utility values, fuzzy preference information, linguistic term vector, selected subset, fuzzy selected subset, pairwise comparison. Other format of preference information is not considered, for example, the interval values. The future work will take into consideration other formats of preference information, for example, the interval values.

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Table 1. Ranking value vectors of alternatives derived from the experts' preference

Experts	Ranking value vector of alternatives derived from the experts' preference
e_1	(0.2222, 0.3333, 0.2778, 0.1667)
e_2	(0.2749, 0.3096, 0.2526, 0.1629)
e_3	(0.1929, 0.2544, 0.2544, 0.2983)
e_4	(0.2167, 0.2750, 0.2917, 0.2167)
e_5	(0.1667, 0.1667, 0.3333, 0.3333)
e_6	(0.1875, 0.2800, 0.1875, 0.3450)
e_7	(0.1715, 0.3144, 0.2364, 0.2781)
Ranking value vector of alternatives from the social preference: (0.2067, 0.2870, 0.2576, 0.2487)	

Table 2. Consensus between the individual expert's preference

	e_1	e_2	e_3	e_4	e_5	e_6	e_6
e_1	1	0.8774	0.6948	0.8484	0.5376	0.6023	0.7478
e_2		1	0.6751	0.8145	0.5158	0.5928	0.6991
e_3			1	0.8126	0.7616	0.8347	0.8347
e_4				1	0.6740	0.6768	0.8005
e_5					1	0.6496	0.6503
e_6						1	0.8239
e_7							1

Table 3. Consensus between individual expert's preference and the social one

	the social preference
e_1' preference	0.8107
e_2' preference	0.7812
e_3' preference	0.8793
e_4' preference	0.9023
e_5' preference	0.6772
e_6' preference	0.7664
e_7' preference	0.8880
The group consensus index	0.815